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The algebraic chromatic splitting conjecture for Noetherian ring spectra. (English)

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Summary: We formulate a version of Hopkins' chromatic splitting conjecture for an arbitrary structured ring spectrum R , and prove it whenever π_*R is Noetherian. As an application, these results provide a new local-to-global principle in the modular representation theory of finite groups.

MSC:

55P Homotopy theory

Full Text: [DOI](#)

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