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Theoretical analysis and numerical simulation for a hyperbolic equation with Dirichlet and acoustic boundary conditions. (English) Zbl 1415.35175

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Summary: This paper is concerned with a theoretical and numerical study for the initial-boundary value problem for a linear hyperbolic equation with variable coefficient and acoustic boundary conditions. On the theoretical results, we prove the existence and uniqueness of global solutions, and the uniform stability of the total energy. Numerical simulations using the finite element method associated with the finite difference method are employed, for one-dimensional and two-dimensional cases, to validate the theoretical results. In addition, numerically the uniform decay rate for energy and the order of convergence of the approximate solution are also shown.

MSC:

35L20	Initial-boundary value problems for second-order hyperbolic equations
65M60	Finite element, Rayleigh-Ritz and Galerkin methods for initial value and initial-boundary value problems involving PDEs
65M06	Finite difference methods for initial value and initial-boundary value problems involving PDEs
35A01	Existence problems for PDEs: global existence, local existence, non-existence
35A02	Uniqueness problems for PDEs: global uniqueness, local uniqueness, non-uniqueness

Keywords:

energy decay; order of convergence of the approximate solution

Full Text: DOI

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