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Grinberg's criterion. (English) [Zbl 1400.05138](#)
Eur. J. Comb. 75, 32-42 (2019).

Summary: We generalize Grinberg's Hamiltonicity criterion for planar graphs. To this end, we first prove a technical theorem for embedded graphs. As a special case of a corollary of this theorem we obtain Zaks' extension of Grinberg's criterion (which encompasses earlier work of *K. R. Gehner* [*Networks* 6, 131–138 (1976; [Zbl 0354.05039](#))] and *Y. Shimamoto* [*J. Comb. Theory, Ser. B* 24, 169–180 (1978; [Zbl 0395.05052](#))]), but the result also implies Grinberg's formula in its original form in a much broader context. Further implications are a short proof for a slightly strengthened criterion of Lewis bounding the length of a shortest closed walk from below as well as a generalization of a theorem due to *J. A. Bondy* and *R. Häggkvist* [*Aequationes Math.* 22, 42–45 (1981; [Zbl 0464.05037](#))].

MSC:

[05C45](#) Eulerian and Hamiltonian graphs

[05C10](#) Planar graphs; geometric and topological aspects of graph theory

Keywords:

[Grinberg's Hamiltonicity criterion](#)

Full Text: [DOI](#)

References:

- [1] Berge, C., (*Graphs and Hypergraphs*, North-Holland Math. Library, vol. 6, (1973))
- [2] Biggs, N. L.; Lloyd, E. K.; Wilson, R. J., *Graph theory, 1736-1936*, (1976), Clarendon Press Oxford · [Zbl 0335.05101](#)
- [3] Bondy, J. A.; Häggkvist, R., Edge-disjoint Hamilton cycles in 4-regular planar graphs, *Aequationes Math.*, 22, 42-45, (1981) · [Zbl 0464.05037](#)
- [4] Bondy, J. A.; Murty, U. S.R., (*Graph Theory*, Graduate Texts in Mathematics, vol. 244, (2008), Springer) · [Zbl 1134.05001](#)
- [5] Chia, G. L.; Thomassen, C., Grinberg's criterion applied to some non-planar graphs, *Ars Combin.*, 100, 3-7, (2011) · [Zbl 1265.05342](#)
- [6] Gehner, K. R., A necessary condition for the existence of a circuit of any specified length, *Networks*, 6, 131-138, (1976) · [Zbl 0354.05039](#)
- [7] Goodman, S. E.; Hedetniemi, S. T., On Hamiltonian walks in graphs, *SIAM J. Comput.*, 3, 214-221, (1974) · [Zbl 0269.05113](#)
- [8] Grinberg, E. J., Plane homogeneous graphs of degree three without Hamiltonian circuits, *Latvian Math. Yearbook*, 4, 51-58, (1968), (in Russian). English translation by D. Zeps available at: <http://www.ltn.lv/~dainize/MathPages/Grinberg.eng.article.pdf> · [Zbl 0185.27901](#)
- [9] Holton, D. A.; McKay, B. D., The smallest non-Hamiltonian 3-connected cubic planar graphs have 38 vertices, *J. Combin. Theory Ser. B*, 45, 305-319, (1988) · [Zbl 0607.05051](#)
- [10] Honsberger, R., *Mathematical gems I, dolciani mathematical expositions no. 1*, (1974), Mathematical Association of America
- [11] Jooyandeh, M.; McKay, B. D.; Östergård, P. R.J.; Pettersson, V. H.; Zamfirescu, C. T., Planar Hypohamiltonian graphs on 40 vertices, *J. Graph Theory*, 84, 121-133, (2017) · [Zbl 1356.05029](#)
- [12] Kirkman, T. P., Question 6610, solution by the proposer, *Math. Quest. Solut. Educ. Times*, 35, 112-116, (1881)
- [13] T.M. Lewis, On the Hamiltonian number of a plane graph, *Discuss. Math. Graph Theory* (in press). <http://dx.doi.org/10.7151/dmgt.2084>, arXiv:1508.06892 [math.CO].
- [14] Mohar, B.; Thomassen, C., *Graphs on surfaces*, (2001), Johns Hopkins University Press Baltimore, MD · [Zbl 0979.05002](#)
- [15] Sachs, H., Ein von kozyrev und grinberg angegebener nicht-hamiltonischer kubischer planarer graph. beiträge zur graphentheorie, 127-130, (1968), Teubner Leipzig, (in German)
- [16] Shimamoto, Y., On an extension of the grinberg theorem, *J. Combin. Theory Ser. B*, 24, 169-180, (1978) · [Zbl 0395.05052](#)
- [17] Thomassen, C., Planar and infinite Hypohamiltonian and hypotraceable graphs, *Discrete Math.*, 14, 4, 377-389, (1976) · [Zbl 0322.05130](#)
- [18] Tutte, W. T., On Hamiltonian circuits, *J. Lond. Math. Soc.*, 21, 98-101, (1946) · [Zbl 0061.41306](#)

- [19] Tutte, W. T., Non-Hamiltonian planar maps, (Read, R., Graph Theory and Computing, (1972), Academic Press New York), 295-301 · [Zbl 0247.05112](#)
- [20] West, D. B., Introduction to graph theory, (2001), Prentice Hall
- [21] Wiener, G., New constructions of Hypohamiltonian and hypotraceable graphs, J. Graph Theory, 87, 526-535, (2018) · [Zbl 1386.05102](#)
- [22] Zaks, J., Extending an extension of grinberg's theorem, J. Combin. Theory Ser. B, 32, 95-98, (1982) · [Zbl 0485.05036](#)

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