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Optimal Hardy-Littlewood inequalities uniformly bounded by a universal constant. (English)

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The original Hardy-Littlewood inequality, published in [*G. H. Hardy* and *J. E. Littlewood*, Q. J. Math., Oxf. Ser. 5, 241–254 (1934; JFM 60.0335.01)], relates some ℓ_r-norm of the coefficients of a bilinear form on $\mathbb{C}^n \times \mathbb{C}^n$ with the supremum on $B_{\ell_p^n} \times B_{\ell_q^n}$. This was later extended to m-linear forms. This article contributes in this direction. Given $1 < p_1, \dots, p_m \leq \infty$, denote $\frac{1}{\mathbf{p}} := \frac{1}{p_1} + \cdots + \frac{1}{p_m}$ (here we use the convention $\frac{1}{\infty} = 0$). If $\frac{1}{2} \leq \frac{1}{\mathbf{p}} < 1$, then, for every $n \in \mathbb{N}$ and every m-linear $T : \mathbb{C}^n \times \cdots \times \mathbb{C}^n \to \mathbb{C}$, we have

$$\left(\sum_{i_1,\dots,i_m=1}^{n} |T(e_{i_1}, \dots, e_{i_m})|^{\frac{1}{1-\frac{1}{\mathbf{p}}}} \right)^{1-\frac{1}{\mathbf{p}}} \leq 2^{(m-1)\left(1-\frac{1}{\mathbf{p}}\right)} \sup_{\substack{\|x_j\|_{p_j}<1 \\ j=1,\dots,m}} |T(x_1, \dots, x_m)|.$$

Some other similar results, involving mixed sums, are also given.

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MSC:

46G25 (Spaces of) multilinear mappings, polynomials
47H60 Multilinear and polynomial operators

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