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Moment measures and stability for Gaussian inequalities. (English) Zbl 1413.28025

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Summary: Let γ be the standard Gaussian measure on $\mathbb{R}^n$ and let $\mathcal{P}_\gamma$ be the space of probability measures that are absolutely continuous with respect to γ. We study lower bounds for the functional $\mathcal{F}_\gamma(\mu) = Ent(\mu) - \frac{1}{2}W_2^2(\mu,\nu)$, where $\mu \in \mathcal{P}_\gamma$, $\nu \in \mathcal{P}_\gamma$, $Ent(\mu) = \int \log(\frac{\mu}{\gamma})d\mu$ is the relative Gaussian entropy, and W_2 is the quadratic Kantorovich distance. The minimizers of $\mathcal{F}_\gamma$ are solutions to a dimension-free Gaussian analogue of the (real) Kähler-Einstein equation. We show that $\mathcal{F}_\gamma(\mu)$ is bounded from below under the assumption that the Gaussian Fisher information of ν is finite and prove a priori estimates for the minimizers. Our approach relies on certain stability estimates for the Gaussian log-Sobolev and Talagrand transportation inequalities.

MSC:

28C20 Set functions and measures and integrals in infinite-dimensional spaces (Wiener measure, Gaussian measure, etc.)
58E99 Variational problems in infinite-dimensional spaces
60H07 Stochastic calculus of variations and the Malliavin calculus

Keywords:

Gaussian inequalities; optimal transportation; Kähler-Einstein equation; moment measure

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