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Le spectre marqué des longueurs des surfaces à courbure négative. (The spectrum marked by lengths of surfaces with negative curvature). (French) [Zbl 0699.58018](#)

Ann. Math. (2) 131, No. 1, 151-162 (1990).

Let S be an orientable closed surface with the genus ≥ 2 . It is known that for a Riemannian metric m with negative curvature any nontrivial conjugate class $\langle \gamma \rangle$ can be represented by a unique geodesic of m . Consider $\ell(\gamma)$ the length of such geodesic. If \mathcal{C} represents the set of the conjugate classes of the fundamental group $\pi_1(S)$ one defines the length's spectrum of the metric m by the element $(\ell(\gamma))_{\gamma \in \mathcal{C}}$ of the direct product $\mathbb{R}^{\mathcal{C}}$. Considering $\mathcal{M}(S)$ the space of the metrics m with negative curvature, one obtains the function $\mathcal{L} : \mathcal{M}(S) \rightarrow \mathbb{R}^{\mathcal{C}}$ defined by $m \mapsto (\ell(\gamma))_{\gamma \in \mathcal{C}}$.

The main purpose of this paper is the proof of the following result: the function \mathcal{L} is one-to-one, i.e. the metrics with negative curvature on S are determined, via an isometry, by their lengths spectrum.

Remark that this result was conjectured by *K. Burns* and *A. Katok* [*Ergodic Theory Dyn. Syst.* 5, 307-317 (1985; [Zbl 0572.58019](#))] and the problem of injectivity of the function \mathcal{L} appears in *V. Guillemin* and *D. Kazhdan* [*Topology* 19, 301-312 (1980; [Zbl 0465.58027](#))] and *A. Katok* [*Ergodic Theory Dyn. Syst.* 8, 139-152 (1988; [Zbl 0668.58042](#))].

Reviewer: [D.Andrica](#)

MSC:

- [58C40](#) Spectral theory; eigenvalue problems on manifolds
- [58D17](#) Manifolds of metrics (especially Riemannian)
- [58E07](#) Variational problems in abstract bifurcation theory in infinite-dimensional spaces
- [58J50](#) Spectral problems; spectral geometry; scattering theory on manifolds
- [53C20](#) Global Riemannian geometry, including pinching

Cited in **9** Reviews
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Keywords:

[metric with negative curvature](#); [spectrum marked by lengths](#); [Riemannian metric](#); [geodesic](#); [isometry](#)

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