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Stirling numbers in braid matroid Kazhdan-Lusztig polynomials. (English) Zbl 1402.05031
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Summary: Restricted Whitney numbers of the first kind appear in the combinatorial recursion for the matroid Kazhdan-Lusztig polynomials. In the special case of braid matroids (the matroid associated to the partition lattice, the complete graph, the type A Coxeter arrangement and the symmetric group) these restricted Whitney numbers are Stirling numbers of the first kind. We use this observation to obtain a formula for the coefficients of the Kazhdan-Lusztig polynomials for braid matroids in terms of sums of products of Stirling numbers of the first kind. This results in new identities between Stirling numbers of the first kind and Stirling numbers of the second kind, as well as a non-recursive formula for the braid matroid Kazhdan-Lusztig polynomials.

MSC:

- 05B35 Combinatorial aspects of matroids and geometric lattices
- 05A15 Exact enumeration problems, generating functions
- 05E10 Combinatorial aspects of representation theory
- 52C35 Arrangements of points, flats, hyperplanes (aspects of discrete geometry)
- 11B73 Bell and Stirling numbers

Cited in 4 Documents

Keywords:

Restricted Whitney numbers of the first kind; braid matroids

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