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Algebraic chromatic homotopy theory for BP_*BP-comodules. (English) Zbl 1412.55005

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The Adams-Novikov spectral sequence provides a close connection between stable homotopy theory and the homological algebra of BP_*BP-comodules. History has taught us that the cleanest way to do homological algebra is often to work directly on the derived level. The classical derived category $\mathcal{D}_{BP_*BP}$ of the abelian category of BP_*BP-comodules has unpleasant properties (as e.g. the tensor unit BP_* is not compact in it). Thus the authors replace it by the stable category Stable_{BP_*BP} of BP_*BP-comodules as introduced by Hovey. In their formulation it is defined as the ind-∞-category on the dualizable objects in $\mathcal{D}_{BP_*BP}$.

Besides basic formal properties about stable categories of Hopf algebroids, the results of the authors fall into two classes:

Firstly, they prove algebraic analogues of the Ravenel conjectures: An algebraic nilpotence theorem, an algebraic telescope conjecture and an algebraic chromatic convergence result. The nilpotence result has necessarily to be weaker than its topological counterpart though, essentially because the vanishing curve on the E_∞-page of the Adams-Novikov spectral sequence is much better than on the E_2-term.

Secondly, they give cleaner proofs for some results in the homological algebra of BP_*BP-comodules, which simultaneously generalize them. We mention in particular their version of the chromatic spectral sequence, which is just a Bousfield-Kan spectral sequence associated to the tower of algebraic chromatic localizations. As a consequence of this spectral sequence, they prove in particular a nice comparison result between the BP-based Adams-Novikov spectral sequence for a bounded below spectrum whose BP-homology has finite projective dimension and its E_n-based one.

We remark that due to work of Isaksen, Gheorghe, Wang and Xu the ∞-category Stable_{BP_*BP} embeds into motivic homotopy theory as modules over a certain motivic spectrum $C\tau$. Thus, the algebraic analogues of the Ravenel conjectures in the present paper are promising to play a role in the exploration of chromatic homotopy theory in the motivic setting.

Reviewer: Lennart Meier (Bonn)

MSC:

55N22	Bordism and cobordism theories and formal group laws in algebraic topology	Cited in **1** Document
55U35	Abstract and axiomatic homotopy theory in algebraic topology	
55P60	Localization and completion in homotopy theory	
14D23	Stacks and moduli problems	
14F42	Motivic cohomology; motivic homotopy theory	

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Hopf algebroid; chromatic localization; chromatic spectral sequence

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