Levin, M. B.

Effectivization of a theorem of Koksma. (Russian) Zbl 0701.11022

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A classical theorem due to *J. P. Koksma* (1935) states that, if f(t,n) ($n = 1, 2, 3, \ldots$) are real valued, continuously differentiable functions of t defined on the interval [a,b] such that for any natural numbers m, n (m$\neq n$) the function $f'(t, m) - f'(t, n)$ is monotonous and satisfies $|f'(t, m) - f'(t, n)| \geq K > 0$, where K is a constant independent of t, m and n, then the sequence $\{f(t, n)\}_{n\geq 1}$ is uniformly distributed (mod 1) for almost all $t \in [a, b]$. Here, it is known also that the discrepancy of the sequence $\{f(t, n)\}_n$ for the first P terms is of $\mathcal{O}(P^{-1/2}(\log P)^{5/2+\epsilon})$ for every $\epsilon > 0$ and almost all t [cf. *J. F. Koksma* and *P. Erdős*, Indagationes Math. 11, 299-302 (1949; Zbl 0035.321) and *J. W. S. Cassels*, Proc. Camb. Philos. Soc. 46, 219-225 (1950; Zbl 0035.319)].

The author obtains the following result. Let f(t,n) ($n = 1, 2, 3, \ldots$) be a sequence of functions satisfying the same conditions as specified above. Put $\alpha = a + \sum_{r=1}^{\infty} a_r(q_r h_r)^{-1}$, where $a_r \geq 0$, $q_r > 0$ and $h_r > 0$ are integers defined inductively on r by some complicated formulae, written basically in terms of values of $f'(a, n)$, $f'(b, n)$ and f(t,n). Then the discrepancy of the particular sequence $\{f(\alpha, n)\}_n$ for the first P terms is of $\mathcal{O}(P^{-1/2}(\log P)^4)$. Thus, one may find for instance a sequence $\{\alpha^n\}_n$ which is uniformly distributed (mod 1), with an effectively determined constant $\alpha > 1$.

Reviewer: S.Uchiyama

MSC:

11J71 Distribution modulo one
11K38 Irregularities of distribution, discrepancy
11K06 General theory of distribution modulo 1

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uniform distribution; real valued, continuously differentiable functions; discrepancy