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Existence results for a Michaud fractional, nonlocal, and randomly position structured fragmentation model. (English) Zbl 1407.35206
Math. Probl. Eng. 2014, Article ID 361234, 8 p. (2014).

Summary: Until now, classical models of clusters' fission remain unable to fully explain strange phenomena like the phenomenon of shattering [*R. M. Ziff* and *E. D. McGrady*, "'Shattering' transition in fragmentation", Phys. Rev. Lett. 58, No. 9, 892–895 (1987)] and the sudden appearance of infinitely many particles in some systems having initial finite number of particles. That is why there is a need to extend classical models to models with fractional derivative order and use new and various techniques to analyze them. In this paper, we prove the existence of strongly continuous solution operators for nonlocal fragmentation models with Michaud time derivative of fractional order [*St. G. Samko* et al., Fractional integrals and derivatives: theory and applications. Transl. from the Russian. New York, NY: Gordon and Breach (1993; Zbl 0818.26003)]. We focus on the case where the splitting rate is dependent on size and position and where new particles generating from fragmentation are distributed in space randomly according to some probability density. In the analysis, we make use of the substochastic semigroup theory, the subordination principle for differential equations of fractional order [*J. Prüss*, Evolutionary integral equations and applications. Basel: Birkhäuser Verlag (1993; Zbl 0784.45006); *E. G. Bazhlekova*, Fract. Calc. Appl. Anal. 3, No. 3, 213–230 (2000; Zbl 1041.34046)], the analogy of Hille-Yosida theorem for fractional model [Prüss, loc. cit.], and useful properties of Mittag-Leffler relaxation function [*M. N. Berberan-Santos*, J. Math. Chem. 38, No. 4, 629–635 (2005; Zbl 1101.33015)]. We are then able to show that the solution operator to the full model is positive and contractive.

MSC:

35R11 Fractional partial differential equations
35A01 Existence problems for PDEs: global existence, local existence, non-existence

Full Text: DOI

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