Campillo, A.; Delgado, F.; Núñez, C. A.
Arithmetic of the Arf semigroups and saturations. Applications.  (Spanish. English summary)
Zbl 0702.14022

Summary: For classifying singularities, suitable sets of invariants are required. In the case of curve singularities, the invariants one often uses have a valuative nature, since one has a canonically associated finite set of valuations. Thus, the semigroup of values of a plane curve singularity characterizes Zariski’s equisingularity type, and the semigroup of the Arf closure (resp. the saturation) characterizes the multiplicity sequence of the resolution process (resp. the equisingularity type of a generic plane projection) for space curve singularities. Saturation has been introduced by Zariski and by Pham-Teissier in a different way. One of the authors gave another definition of saturation which is very appropriate to handle the value semigroups arithmetically [A. Campillo, in Singularities, Summer Inst. Arcata/Calif. 1981, Proc. Symp. Pure Math. 40, Part 1, 211-220 (1983; Zbl 0553.14013 and in Singularities, Banach Cent. Publ. 20, 121-137 (1988; see the preceding review)]. As a consequence the semigroups of one dimensional C. M. local rings have a nice structure which is very much simpler than that of the semigroups of plane curves and, even, they have a more natural geometrical interpretation. This nice structure follows from Arf property which is satisfied by the saturated rings. In this communication, we show how the definitions of Arf and saturation can be extended to the relative situation of an arbitrary (local) ring and finitely many discrete valuations of it, in such a way that the actual semigroups also have a nice structure providing reasonable invariants. As an application, one obtains a classification method for singularities having a canonical resolution.

MSC:
14H20 Singularities of curves, local rings
14B05 Singularities in algebraic geometry
13H10 Special types (Cohen-Macaulay, Gorenstein, Buchsbaum, etc.)
14E15 Global theory and resolution of singularities (algebro-geometric aspects)

Keywords:
classifying singularities; space curve singularities; saturation; value semigroups; semigroups of one dimensional C. M. local rings; canonical resolution