Streb, Walter

Zur Struktur nichtkommutativer Ringe. (On the structure of noncommutative rings). (German) Zbl 0702.16022

Math. J. Okayama Univ. 31, 135-140 (1989).

Let $\mathcal{R}$ be a class of rings, and $\mathcal{K}$ the subclass of non- commutative rings in $\mathcal{R}$. If R,T$\in \mathcal{K}$, T is called R-reducing if there exists a finite sequence R_i, $i = 0, ..., n$, of rings in $\mathcal{K}$ such that $R_0 = R$, $R_n = T$ and for $0 \leq i < n$, R_{i+1} is either a subring of R_i or a homomorphic image of R_i. A subclass $\mathcal{K}^*$ of $\mathcal{K}$ is called $\mathcal{K}$-reducing if for each $R \in \mathcal{K}$, there exists an R-reducing $T \in \mathcal{K}^*$. This paper exhibits $\mathcal{K}$-reducing subclasses for various important classes $\mathcal{R}$- for example, the class of all rings, the class of rings with 1, the class of PI-rings.

This work provides a technique for proving commutativity theorems. Specifically, let P be a property defined on rings of the class $\mathcal{R}$, and persisting under taking subrings and homomorphic images. To show commutativity of all rings in $\mathcal{R}$ with property P, one need only show that the rings in a $\mathcal{K}$-reducing subclass of $\mathcal{R}$ fail to have property P.

Reviewer: H.E.Bell

MSC:

16U70 Center, normalizer (invariant elements) (associative rings and algebras)
16D70 Structure and classification for modules, bimodules and ideals (except as in 16Gxx), direct sum decomposition and cancellation in associative algebras)

Keywords:

$\mathcal{K}$-reducing subclasses; commutativity theorems; subrings; homomorphic images