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Nondegeneracy of blow up for semilinear heat equations. (English) Zbl 0703.35020
Commun. Pure Appl. Math. 42, No. 6, 845-884 (1989).

This paper is a rather comprehensive treatment of the blow up of solutions to the equation

$$u_t - \Delta u - |u|^{p-1}u = 0 \text{ in } D \times (0, T),$$

with zero boundary data. Here D is a domain in \mathbb{R}^n , u is scalar-valued, and $p > 1$.

A first set of results is concerned more generally with the solutions of the parabolic differential inequality $|v_t - \Delta v| \leq k(1 + |v|^p)$, $k > 0$ in a cylinder $Q_r = B_r(a) \times (t_1 - r^2, t_1)$, $0 < r \leq 1$. For instance it is proved that if $|v(x, t)| \leq \epsilon(t_1 - t)^{-1/(p-1)}$ in Q_r for some appropriate $\epsilon = \epsilon(k, p, n)$, then (a, t_1) is not a blow up point for v .

Another interesting conclusion regards a sufficient condition for excluding blow up of u at a given point in terms of the smallness of an energy-type functional. The behaviour of solutions near blow up points is characterized by the so-called blow up limit, showing that tending to blow up point (a, T) along parabolas $x = a + y(T - t)^{1/2}$, the limit of $(T - t)^{1/(p-1)}u$ has to be $\pm(p - 1)^{-1/(p-1)}$. In addition, the authors describe the structure of the blow up set and they present some extensions to more general equations.

Reviewer: [A.Fasano](#)

MSC:

[35B40](#) Asymptotic behavior of solutions to PDEs

[35K60](#) Nonlinear initial, boundary and initial-boundary value problems for linear parabolic equations

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