Mitrea, Alexandru-Ioan

On the instability of generalised convergence in mean of Lagrange interpolation processes for the Chebyshev matrix of the first kind. (English) Zbl 0703.41003

In 1981 D. L. Berman gave the surprising result that there exists a function $f \in C([-1,1])$ such that if the polynomial $L_n(f, x)$ of degree $\leq n+1$, interpolates $f$ on the zeros of the Chebyshev polynomial $T_n(x)$ and at $\pm 1$, then

$$\limsup_{n \to \infty} \int_{-1}^{1} |f(x) - L_n(f, x)|^2 \frac{dx}{\sqrt{1-x^2}} = \infty.$$ 

Thus the well-known property of mean convergence of Lagrange interpolates on the zeros of $T_n(x)$ for any $f \in [-1,1]$ is disturbed by adding the nodes $\pm 1$. This does not tell us about the cardinality or density of such singular functions in the space of continuous functions. Recently (*) S. Cobzas and I. Muntean [J. Approximation Theory 31, 138-153 (1981; Zbl 0478.41030)] studied the topological structure of the set of all singular functions for unboundedly divergent approximation methods. Here the author extends the results of Berman in the spirit of the results of (*). His main result is: Let $p \geq 1$ be a real fixed number.

There exists a set $X_0$ of functions superdense in $C[-1,1]$ endowed with the uniform norm such that:

$$\int \limsup_{n \to \infty} \int_{-1}^{1} |f(x) - \tilde{L}_n(f, x)|^p (1-x^2)^{-1/2} dx = +\infty$$

for every $f \in X_0$, where $\tilde{L}_n(f; x)$ is the modified Lagrange interpolant of Berman. (A subset $S$ of a topological space $T$ is called superdense if $S$ is uncountably infinite dense and a $G_\delta$-set in $T$.)

Reviewer: A. Sharma

MSC:

41A05 Interpolation in approximation theory

Keywords:

Chebyshev polynomial; Lagrange interpolates