

Khavkine, Igor; Melati, Alberto; Moretti, Valter

On Wick polynomials of boson fields in locally covariant algebraic QFT. (English)

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Summary: This work presents some results about Wick polynomials of a vector field renormalization in locally covariant algebraic quantum field theory in curved spacetime. General vector fields are pictured as sections of natural vector bundles over globally hyperbolic spacetimes and quantized through the known functorial machinery in terms of local $*$ -algebras. These quantized fields may be defined on spacetimes with given classical background fields, also sections of natural vector bundles, in addition to the Lorentzian metric. The mass and the coupling constants are in particular viewed as background fields. Wick powers of the quantized vector field are axiomatically defined imposing in particular local covariance, scaling properties, and smooth dependence on smooth perturbation of the background fields. A general classification theorem is established for finite renormalization terms (or counterterms) arising when comparing different solutions satisfying the defining axioms of Wick powers. The result is specialized to the case of general tensor fields. In particular, the case of a vector Klein-Gordon field and the case of a scalar field renormalized together with its derivatives are discussed as examples. In each case, a more precise statement about the structure of the counterterms is proved. The finite renormalization terms turn out to be finite-order polynomials tensorially and locally constructed with the background fields and their covariant derivatives whose coefficients are locally smooth functions of polynomial scalar invariants constructed from the so-called marginal subset of the background fields. The notion of local smooth dependence on polynomial scalar invariants is made precise in the text. Our main technical tools are based on the Peetre-Slovák theorem characterizing differential operators and on the classification of smooth invariants on representations of reductive Lie groups.

MSC:

- 83C47 Methods of quantum field theory in general relativity and gravitational theory
- 81T20 Quantum field theory on curved space or space-time backgrounds
- 81Q05 Closed and approximate solutions to the Schrödinger, Dirac, Klein-Gordon and other equations of quantum mechanics
- 53Z05 Applications of differential geometry to physics
- 81T16 Nonperturbative methods of renormalization applied to problems in quantum field theory
- 22D25 C^* -algebras and W^* -algebras in relation to group representations

Cited in 4 Documents

Keywords:

Wick polynomials; vector field renormalization; locally covariant algebraic quantum field theory; curved spacetime; local $*$ -algebras; Klein-Gordon field

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