Muhly, Paul S.; Saito, Kichi-Suke; Solel, Baruch
Coordinates for triangular operator algebras. II. (English) Zbl 0704.47034
Pac. J. Math. 137, No. 2, 335-369 (1989).

Write $\mathcal{M}$ for the full algebra of complex $n \times n$ matrices, $\mathcal{A}$ for its subalgebra of diagonal matrices, and $\mathcal{P}$ for the intermediate algebra of upper-triangular matrices. Fix δ in $\mathcal{A}$ with strictly decreasing real diagonal entries and define a group of automorphisms of $\mathcal{M}$ by $\alpha_t(a) = e^{it\delta} a e^{-it\delta}$. Then $\mathcal{P}$ can be recovered as the "analytic algebra" consisting of those a for which the maps $t \to \rho \circ \alpha_t(a)$ belong to the classical Hardy space $H^\infty(R)$ for each $\rho \in \mathcal{M}^*$.

The present paper studies the possibility of generalizing this construction. $\mathcal{M}$ is taken to be a von Neumann algebra, $\mathcal{A}$ a Cartan subalgebra of $\mathcal{M}$, and $\mathcal{P}$ a maximal subdiagonal algebra lying between them; the question is when $\mathcal{P}$ can be recovered as the analytic algebra corresponding to some one-parameter automorphism group of $\mathcal{M}$. The answer is given in terms of the representation studied in Part I of this paper [Ann. Math. 127, 245-278 (1988; Zbl 0649.47036)]: members of $\mathcal{M}$ are represented as matrices indexed by an equivalence relation R on a measure space, and the members of $\mathcal{P}$ are supported on some partial order $P \subseteq R$. Then $\mathcal{P}$ is an analytic algebra iff $P = d^{-1}(R^+)$ for some cocycle d. This is always the case when $\mathcal{M}$ is Type I, but the authors construct and examine several explicit examples where it fails.

Reviewer: E.Azoff

MSC:

47L30	Abstract operator algebras on Hilbert spaces	Cited in **2** Documents
46L10	General theory of von Neumann algebras	
46L40	Automorphisms of selfadjoint operator algebras	
47B25	Linear symmetric and selfadjoint operators (unbounded)	
47A66	Quasitriangular and nonquasitriangular, quasidiagonal and nonquasidiagonal linear operators	
46H15	Representations of topological algebras	

Keywords:

algebra of upper-triangular matrices; Hardy space; von Neumann algebra; Cartan subalgebra; maximal subdiagonal algebra; analytic algebra corresponding to some one-parameter automorphism group; cocycle

Full Text: DOI