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Betti numbers of MCM modules over the cone of an elliptic normal curve. (English)

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Summary: We apply Orlov's equivalence to derive formulas for the Betti numbers of maximal Cohen-Macaulay modules over the cone an elliptic curve (E, x) embedded into \mathbb{P}^{n-1} , by the full linear system $|\mathcal{O}(nx)|$, for $n > 3$. The answers are given in terms of recursive sequences. These results are applied to give a criterion of (Co-)Koszulity.

In the last two sections of the paper we apply our methods to study the cases $n = 1, 2$. Geometrically these correspond to the embedding of an elliptic curve into a weighted projective space. The singularities of the corresponding cones are called minimal elliptic. They were studied by K. Saito [13], where he introduced the notation \widetilde{E}_8 for $n = 1$, \widetilde{E}_7 for $n = 2$ and \widetilde{E}_6 for the cone over a smooth cubic, that is, for the case $n = 3$. For the singularities \widetilde{E}_7 and \widetilde{E}_8 we obtain formulas for the Betti numbers and the numerical invariants of MCM modules analogous to the case of a plane cubic.

MSC:

- 14H52 Elliptic curves
- 13C14 Cohen-Macaulay modules
- 13D02 Syzygies, resolutions, complexes and commutative rings
- 14F05 Sheaves, derived categories of sheaves, etc. (MSC2010)

Keywords:

elliptic curves; maximal Cohen-Macaulay modules; ulrich modules; Koszul modules

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