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The sigma function for trigonal cyclic curves. (English) Zbl 1441.14145

Let. Math. Phys. 109, No. 2, 423-447 (2019).

The Weierstrass semigroup of a complex pointed curve (X, p) is the complement of the Weierstrass gap sequence at p . It determines the Weierstrass normal form of X with respect to p .

In the present article, the authors use the Weierstrass normal form of a given pointed trigonal cyclic curve (X, p) to obtain a number of detailed results about differentials and the sigma function. For instance, they obtain an explicit description of a basis for the vector space of differential one forms and thus coordinates for the canonical embedding. They also obtain detailed results which apply to differentials of the second and third kinds.

The authors then turn to the study of the Jacobian of X . As one result, they study the sigma function and present a solution to the Jacobi inversion problem. Many of the authors methods are sufficiently general so as to apply to the more general case of k -gonal cyclic covers of \mathbb{P}^1 .

Reviewer: [Nathan Grieve \(Ottawa\)](#)

MSC:

[14K25](#) Theta functions and abelian varieties

[14H40](#) Jacobians, Prym varieties

[14H55](#) Riemann surfaces; Weierstrass points; gap sequences

Cited in 1 Document

Keywords:

[trigonal cyclic curve](#); [Weierstrass semigroup](#); [sigma function](#)

Full Text: [DOI](#)

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