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The number of zeros of unilateral polynomials over coquaternions revisited. (English)

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Summary: The literature on quaternionic polynomials and, in particular, on methods for finding and classifying their zero sets, is fast developing and reveals a growing interest in this subject. In contrast, polynomials defined over the algebra of coquaternions have received very little attention from researchers. One of the few exceptions is the very recent paper by *D. Janovská* and *G. Opfer* [ETNA, Electron. Trans. Numer. Anal. 46, 55–70 (2017; Zbl 1368.65069)], where, among other results, we can find a first attempt to prove that a unilateral coquaternionic polynomial of degree n has, at most, $n(2n - 1)$ zeros. In this paper we present a full proof of this result, using a totally different and, from our point of view, much simpler approach. Also, we give a complete characterization of the zero sets of such polynomials and present a new result giving conditions which guarantee the existence of a special type of zeros. An algorithm to compute and classify all the zeros of a coquaternionic polynomial is proposed and several numerical examples are carefully constructed.

MSC:

15A66 Clifford algebras, spinors

12E05 Polynomials in general fields (irreducibility, etc.)

65H04 Numerical computation of roots of polynomial equations

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coquaternions; coquaternionic polynomials; companion polynomial; admissible classes

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