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Divergence free polar wavelets for the analysis and representation of fluid flows. (English)

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Summary: We present a Parseval tight wavelet frame for the representation and analysis of velocity vector fields of incompressible fluids. Our wavelets have closed form expressions in the frequency and spatial domains, are divergence free in the ideal, analytic sense, have a multi-resolution structure and fast transforms, and an intuitive correspondence to common flow phenomena. Our construction also allows for well defined directional selectivity, e.g. to model the behavior of divergence free vector fields in the vicinity of boundaries or to represent highly directional features like in a von Kármán vortex street. We demonstrate the practicality and efficiency of our construction by analyzing the representation of different divergence free vector fields in our wavelets.

MSC:

42C40 Nontrigonometric harmonic analysis involving wavelets and other special systems
76B99 Incompressible inviscid fluids

Keywords:

divergence freedom; wavelets; tight frames

Software:

Steerable pyramid

Full Text: DOI

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