Billey, Sara C.; **Holroyd, Alexander E.**; **Young, Benjamin J.**
A bijective proof of Macdonald's reduced word formula. (English) Zbl 1409.05024
Algebr. Comb. 2, No. 2, 217-248 (2019).

Summary: We give a bijective proof of Macdonald's reduced word identity using pipe dreams and Little's bumping algorithm. This proof extends to a principal specialization due to *S. Fomin* and *R. P. Stanley* [Adv. Math. 103, No. 2, 196–207 (1994; Zbl 0809.05091)]. Such a proof has been sought for over 20 years. Our bijective tools also allow us to solve a problem posed by *S. Fomin* and *A. N. Kirillov* [J. Algebr. Comb. 6, No. 4, 311–319 (1997; Zbl 0882.05010)] using work of Wachs, Lenart, Serrano and Stump. These results extend earlier work by the third author ["A Markov growth process for Macdonald's distribution on reduced words", Preprint, arXiv:1409.7714] on a Markov process for reduced words of the longest permutation.

MSC:

05A17	Combinatorial aspects of partitions of integers
05A15	Exact enumeration problems, generating functions
05E10	Combinatorial aspects of representation theory
05E05	Symmetric functions and generalizations
14M15	Grassmannians, Schubert varieties, flag manifolds
60J99	Markov processes

Keywords:

Young tableaux; Ferrers shape; reduced words; identity; Stanley's formula; Macdonald's formula; Schubert polynomials; enumeration of plane partitions; permutations; shapes

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