Kamienny, S.
On $J_1(p)$ and the kernel of the Eisenstein ideal. (English) Zbl 0705.14025

The covering map $\pi : X_1(p) \to X_0(p)$ between the modular curves corresponding to the congruence subgroups of level $p$ defines a natural map on their Jacobian varieties $\pi_* : J_0(p) \to J_1(p)$. Let $J$ be the quotient of $J_1(p)$ by the image of $J_0(p)$ and let $T$ denote the Hecke algebra acting on $J$ by Albanese functoriality. Let $\epsilon$ be an even character of conductor $p$, where $p$ denotes a prime $\geq 13$. The author proves that the group scheme $J_p[I_\epsilon]$ over $\text{Spec}(\mathbb{Z}[1/p])$ given by the kernel of the Eisenstein ideal $I_\epsilon$ acting on the $p$-divisible group $J_p$ of $J$ is admissible and not pure; moreover, the splitting field of this group scheme is an everywhere unramified $p$-extension of $\mathbb{Q}(\zeta_p)$ on which the Galois group $\text{Gal}(\mathbb{Q}(\zeta_p)/\mathbb{Q})$ acts via the character $\chi^{-1}\epsilon^{-1}$.

Let $C_p(\epsilon)$ be the $\epsilon$-eigenspace under the action of the Galois group of the cover $X_1(p)/X_0(p)$ on the $p$-part of the projection to $J$ of the cuspidal class group of $J_1(p)$. Since the ideal $I_\epsilon$ annihilates $C_p(\epsilon)$, the author can give as a corollary a more constructive proof of Ribet’s theorem on the converse to Herbrand’s criterion.

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14H40 Jacobians, Prym varieties

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