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Two remarks on homogeneous varieties of representations of groups. (Russian)

Zbl 0705.20005
Latv. Mat. Ezheg. 33, 84-91 (1989).

Let F be the free group with basis $X = \{x_i |\ i \in \mathbb{N}\}$ and kF be its group ring over a field k, char $k \neq 2, 3$. The Magnus embedding of kF into the algebra of formal power series $k[[z_i |\ i \in \mathbb{N}]]$ is given by $x_i \mapsto 1 + z_i$ (i$\in \mathbb{N}$). To any $u = u(x_1, ..., x_m) \in kF$ there corresponds the series $\hat{u}$ which can be uniquely represented as the sum $\sum_{n=0}^{\infty} u_{<n>}$ of its homogeneous components, the nth component $u_{<n>}$ being a finite sum of monomials $\lambda_{(i)} z_{i_1} ... z_{i_n}$ of degree n with coefficients $\lambda_{(i)}$ in k.

A variety $\mathfrak{X}$ of group representations (over k) is said to be homogeneous iff from $\hat{u}$ being an identity for $\mathfrak{X}$ it follows that all its homogeneous components are also identities for $\mathfrak{X}$. It appears that the variety $\mathfrak{S}_5$ of group representations, given by the identities $z_1 z_2 z_3 z_4 z_5$ and $2z_1 z_2 z_1 - z_1 z_2^2 z_1$ is not homogeneous [the author, Sib. Mat. Zh. 29, 35-47 (1988; Zbl 0661.20003)]. Also, the variety of 3-unipotent representations of groups, given by z^3 is not homogeneous [Th. 2 in the present paper].

To formulate the main concern in this paper, one more notion is needed. An element $u \in kF$ is said to be a special word of type (n,m,k) iff the following holds: (1) $\{z_1, ..., z_m\}$ serves as the support for every monomial in $\hat{z}$ (2) t is minimal with $u_{<t>}$ equals k and (3) $u_{<i>} \notin \hat{Id}(u, \Delta^n)$ for some i, with $Id(u, \Delta^n)$ being the verbal ideal in kF containing u and $\Delta^n(k, F)$. The result about $\mathfrak{S}_5$ cited above follows from the fact that $u(x_1, x_2)$ with $\hat{u} = 2z_1 z_2 z_1 - z_1 z_2^2 z_1$ is a special word of type (5,2,3). The author proves (Th.1): there exists no special word of type $< (5, 2, 3)$ with " $<''$ being the lexicographical order (from the left).

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MSC:

20C15 Ordinary representations and characters
20C07 Group rings of infinite groups and their modules (group-theoretic aspects)
16W60 Valuations, completions, formal power series and related constructions (associative rings and algebras)
16R10 T-ideals, identities, varieties of associative rings and algebras
20E10 Quasivarieties and varieties of groups

Keywords:

homogeneous variety of group representations; free group; group ring; Magnus embedding; algebra of formal power series; homogeneous components; special word