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Induced $\mathcal{D}$-modules and differential complexes. (English) Zbl 0705.32005

Bull. Soc. Math. Fr. 117, No. 3, 361-387 (1989).

Let $X \to Y$ be a proper morphism of complex manifolds or of smooth algebraic varieties. It is known that one can define for any bounded complex M^* of $\mathcal{D}_X$-modules with coherent cohomologies a duality isomorphism $f_*\mathbb{D}M^* \to \mathbb{D}f_*M^*$. In this formula f_* is the direct image of $\mathcal{D}$-modules and $\mathbb{D}$ is the dual functor $\mathbb{R}Hom_{\mathcal{D}_X}(M^*, \mathcal{D}_X(d_x))$ (case of right $\mathcal{D}_X$-modules).

The aim of this paper is, in particular, to give a new proof of this result, based on the notion of induced $\mathcal{D}_X$-module introduced by the author. These are the $\mathcal{D}_X$-modules of the type $L \otimes_{\mathcal{O}_X} \mathcal{D}_X$, for some $\mathcal{O}_X$-module L. Applying the functor $\otimes_{\mathcal{D}_X}\mathcal{O}_X$ one gets the notion of differential morphisms $L \to L'$ which are $\mathcal{D}_X$ linear map induced by $\mathcal{D}_X$-linear morphisms $L \otimes \mathcal{D}_X \to L' \otimes \mathcal{D}_X$. The author introduces also the notion of differential morphisms of finite order p which correspond to maps $L \to L' \otimes F_p\mathcal{D}_X$. In § 1, he proves some equivalences of categories quite useful for the next sections:

The category $D^b(\mathcal{O}_X, Diff)$ (resp. $D^b(\mathcal{O}_X, Diff)^f$) of complex of $\mathcal{O}_X$-modules (resp. with finite order differential), is equivalent to $D^b(\mathcal{O}_X)$. In the first derived category are inverted the D-quasi-isomorphism, i.e. those morphism of complexes which via $\otimes_{\mathcal{O}_X}\mathcal{D}_X$ induce quasi-isomorphisms in $C^b(\mathcal{D}_X)$. Similarly the author defines $D^b(\mathcal{O}_X, Diff)_{coh}$ via this equivalence. In § 2 and § 3, he obtains then his main results: First, definition of dual functors *in* both categories and then proof of the compatibility of $\mathbb{D}$, with the De Rham functor in the holonomic case, and with f^* in general. Finally in the last § 4, the author introduces the concept of diagonal pairing and gives two applications of it, to a definition of the dual functor $\mathbb{D}$ first and then to a simplification in the proof of the Riemann Hilbert correspondence.

Reviewer: J.M.Granger

MSC:

32C37 Duality theorems for analytic spaces
14F10 Differentials and other special sheaves; D-modules; Bernstein-Sato ideals and polynomials
32C38 Sheaves of differential operators and their modules, D-modules
14F40 de Rham cohomology and algebraic geometry

Keywords:

differential complexes; direct image; duality; $\mathcal{D}$-modules

Full Text: DOI Numdam EuDML

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