Given a based map $f: S^n \to X$, the generalized reduced cohomology theory $E$ provides us with the notion of the codegree $cd(f; E^m)$ of $f$, defined to be the cardinal number of the cokernel of the composite

$$E^{m+n}(X) \xrightarrow{f^*} E^{m+n}(S^n) \to E^m(S^0)/Tor.$$ 

The author’s main concern is $cd(f) = cd(f; \pi^0_3)$ where $\pi^0_3$ is the stable cohomotopy functor, especially the codegree $cd(G) = cd(f)$ of a map $f$ generating $\pi^0_3(G) \cong \mathbb{Z}$ for a simply connected simple Lie group $G$. In this connection his main results are as follows. 1) $cd_p(G)$, the exponent of the prime $p$ in $cd(G)$, is 0 if and only if $G$ is $p$-regular. 2) $cd_p(G) = 1$ if $G$ is not $p$-regular but quasi $p$-regular. 3) $cd_p(SU(n)) = \max\{i; p^i < n\}$ if $p > 2$ and $n \geq 3$; $cd_p(SO(2n+1)) = cd_p(Sp(n))$ for $p > 2$ and $n \geq 1$; $cd_p(SO(2n)) = cd_p(Spin(2n)) = cd_p(Spin(2n-1))$ for $p > 2$ and $n \geq 3$; $cd(Sp(n)) = cd(SU(2n))$. 4) $cd(E_8) = cd(E_7) = cd_5(E_7) = 1; cd(G_2) = cd(Spin(7)) = cd(Spin(8)) = cd(SO(7)) = 2 \cdot cd(SU(7)) = 2^3 \cdot 3 \cdot 5$ and so on. These are deduced with the help of the work of M. C. Crabb and K. Knapp [Math. Ann. 282, 395-422 (1988; Zbl 0627.57026)] by using the fact that the codegree coincides with that of a canonical bundle over a projective space. Some of the codegrees for small $p$ are left undetermined.

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MSC:
55Q55 Cohomotopy groups
57T10 Homology and cohomology of Lie groups
55N15 Topological $K$-theory

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cohomology theory; codegree; stable cohomotopy; Lie group; exponent; $p$-regular; canonical bundle; projective space