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On  $p$ -adic  $L$ -functions. (English) [Zbl 0706.11064](#)

Sémin. Bourbaki, Vol. 31, 41e année (1988/1989), Exp. No. 701, Astérisque 177-178, 33-59 (1989).

The principal conjecture of this article asserts that there is a two-variable  $p$ -adic  $L$ -function interpolating special values of a complex motivic  $L$ -function. To be more precise, let  $H$  and  $D$  denote, respectively, the Galois groups of the field generated by all  $p$ -power roots of unity and of its maximal real subfield, and let  $\psi : H \rightarrow \mathbb{Z}_p^*$  be the canonical isomorphism defined by the action of  $H$  on the group of all  $p$ -power roots of unity. Then, according to the author's conjecture, there is a unique "pseudo-measure"  $\mu$  on  $D$  such that  $L(\chi, n) := \int_D \psi \chi^n d\mu$  is equal to a properly normalised special value of a certain motivic  $L$ -function; here  $n \in \mathbb{Z}$ , and  $\chi$  ranges over all the Dirichlet characters of  $p$ -power conductor, with  $\chi(-1) = (-1)^n$ .

According to the author, this conjecture is basically taken from his joint work with *B. Perrin-Riou* [cf. Algebraic number theory - in honour of K. Iwasawa, Proc. Workshop Iwasawa Theory Spec. Values  $L$ -Funct., Berkeley/CA (USA) 1987, Adv. Stud. Pure Math. 17, 23–55 (1989; [Zbl 0783.11039](#))] however, P. Deligne "pointed out that there is a much more elegant and succinct way of expressing our conjectures using the local  $\varepsilon$ -factors of the motive".

The relevant definitions and results concerning  $p$ -adic analogues of Dirichlet  $L$ -functions and complex motivic  $L$ -functions are reviewed in some detail. In the introduction, the author states a theorem from his forthcoming joint work with M. Flach. This theorem relates the value at  $s = 1$  of the complex  $L$ -function attached to the motive, which is the symmetric square of an elliptic curve, with Kummer's theory.

For the entire collection see [[Zbl 0691.00001](#)].

Reviewer: [B. Z. Moroz](#)

#### MSC:

- [11G40](#)  $L$ -functions of varieties over global fields; Birch-Swinnerton-Dyer conjecture
- [11R23](#) Iwasawa theory
- [11S40](#) Zeta functions and  $L$ -functions
- [14G10](#) Zeta functions and related questions in algebraic geometry (e.g., Birch-Swinnerton-Dyer conjecture)

Cited in **2** Reviews  
Cited in **6** Documents

#### Keywords:

cyclotomic fields; two-variable  $p$ -adic  $L$ -function; special values; complex motivic  $L$ -function; pseudo-measure

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