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Optimality of the Paterson-Stockmeyer method for evaluating matrix polynomials and rational matrix functions. (English) Zbl 1431.65062

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Summary: Many state-of-the-art algorithms reduce the computation of transcendental matrix functions to the evaluation of polynomial or rational approximants at a matrix argument. This task can be accomplished efficiently by resorting to the Paterson-Stockmeyer method, an evaluation scheme originally developed for polynomials of matrices that extends quite naturally to rational functions. An important feature of this technique is that the number of matrix multiplications required to evaluate an approximant of order n grows slower than n itself, with the result that different approximants yield the same asymptotic computational cost. We analyze the number of matrix multiplications required by the Paterson-Stockmeyer method and by two widely used generalizations, one for evaluating diagonal Padé approximants of general functions and one specifically tailored to those of the exponential. In all the three cases, we identify the approximants of maximum order for any given computational cost.

MSC:

65F60 Numerical computation of matrix exponential and similar matrix functions

15A16 Matrix exponential and similar functions of matrices

Keywords:

Paterson-Stockmeyer method; polynomial evaluation; matrix polynomial; matrix rational function; matrix function

Software:

MATLAB expm; mftoolbox

Full Text: DOI

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