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A diagonal embedding theorem for function spaces with dominating mixed smoothness properties. (English) Zbl 0707.46020

Approximation and function spaces, Proc. 27th Semest., Warsaw/Pol. 1986, Banach Cent. Publ. 22, 475-486 (1989).

[For the entire collection see Zbl 0681.00013.]

Spaces with dominating mixed smoothness properties of Sobolev type were introduced by *S. M. Nikol'skij* [Dokl. Akad. Nauk SSSR 146, 542-545 (1962; Zbl 0196.443)]. The simplest case on the plane $\mathbb{R}^2$ is characterized by the norm $\|f\|+\|\frac{\partial f}{\partial x_1}\|+\|\frac{\partial f}{\partial x_2}\|+\|\frac{\partial^2 f}{\partial x_1 \partial x_2}\|$ where $\|\|$ is the $L_p(\mathbb{R}^2)$ norm.

This paper deals with spaces $b^r_p(\mathbb{R}^2)$ where $r=(r_1,r_2)$, $-\infty<r_j<\infty$, $1\leq p\leq\infty$. The norm is of Besov type and provides dominating mixed smoothness of order r_j with respect to x_j for $j=1,2$. The corresponding trace spaces are the Besov spaces $b^\rho_p(\mathbb{R})$. It is proved that the diagonal mapping T: $\mathrm{f}(\mathrm{x}_1,x_2)\to f(x_1,x_1)$ is a retraction from $b^r_p(\mathbb{R}^2)$ onto $b^\rho_p(\mathbb{R})$ where $\rho=\min(r_1+r_2-1/p,r_1,r_2)$.

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MSC:

46E35 Sobolev spaces and other spaces of "smooth" functions, embedding theorems, trace theorems

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Sobolev space; Spaces with dominating mixed smoothness properties of Sobolev type; trace spaces; Besov spaces; diagonal mapping; retraction