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Convexity methods in Hamiltonian mechanics. (English) Zbl 0707.70003

Ergebnisse der Mathematik und ihrer Grenzgebiete, 3. Folge, 19. Berlin etc.: Springer-Verlag. x, 247 p. DM 148.00 (1990).

This book is written with two purposes: (a) to serve as an introduction to the subject of convex Hamiltonian system, and (b) to serve as a reference for the properties of the index of periodic solutions to convex Hamiltonian systems. The author concentrated on the various aspects of index theory. The index, as the author defines it in this book, is an integer associated with a linear system on a time interval, provided the Hamiltonian is positive definite. Other definitions exist, valid for any quadratic Hamiltonian, but here again the convexity assumption enables to give a purely analytic description, without having to go into the geometry of the symplectic group.

The author has intended Chapter I to be a complete exposition of index theory. So the first three sections of Chapter I contain an account of Krein theory, which will be used in later sections to prove iteration formulas for the index.

Chapter II is a self-contained account of convex analysis, and gives the abstract duality principle which is the setting for the remainder of the book. The last Chapters III, IV and V, contain results on periodic solutions of nonlinear Hamiltonian systems. Chapters III and IV are mostly concerned with nonautonomous problems: the fundamental period is prescribed. The existence of solutions with the given period T (fundamental mode), and also of solutions with higher period kT (subharmonics) is shown.

Chapter V deals with autonomous problems: the energy of the system is prescribed and the periodic solutions (of any period) lying on that energy level are also studied. A very important result is theorem V.3.15, which shows that there are at least two such solutions on each energy level, and infinitely many in general (except in the trivial case where there is just one degree of freedom). Finally, the bibliography is extensive. It contains 159 items among them 23 papers by the author. The proofs are written out with great care and in detail, which makes the book easy to read.

Reviewer: [G.Dimitriu](#)

MSC:

- [70-02](#) Research exposition (monographs, survey articles) pertaining to mechanics of particles and systems
- [58-02](#) Research exposition (monographs, survey articles) pertaining to global analysis
- [70Hxx](#) Hamiltonian and Lagrangian mechanics

Cited in **363** Documents

Keywords:

[convex Hamiltonian system](#); [index theory](#); [symplectic group](#); [autonomous problems](#); [periodic solutions](#)