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On the uniform domination number of a finite simple group. (English) Zbl 07076400

Summary: Let $G$ be a finite simple group. By a theorem of Guralnick and Kantor, $G$ contains a conjugacy class $C$ such that for each nonidentity element $x \in G$, there exists $y \in C$ with $G = \langle x, y \rangle$. Building on this deep result, we introduce a new invariant $\gamma_u(G)$, which we call the uniform domination number of $G$. This is the minimal size of a subset $S$ of conjugate elements such that for each $1 \neq x \in G$, there exists $s \in S$ with $G = \langle x, s \rangle$. (This invariant is closely related to the total domination number of the generating graph of $G$, which explains our choice of terminology.) By the result of Guralnick and Kantor, we have $\gamma_u(G) \leq |C|$ for some conjugacy class $C$ of $G$, and the aim of this paper is to determine close to best possible bounds on $\gamma_u(G)$ for each family of simple groups. For example, we will prove that there are infinitely many nonabelian simple groups $G$ with $\gamma_u(G) = 2$. To do this, we develop a probabilistic approach based on fixed point ratio estimates. We also establish a connection to the theory of bases for permutation groups, which allows us to apply recent results on base sizes for primitive actions of simple groups.

MSC:
20E32 Simple groups
20F05 Generators, relations, and presentations of groups
20E28 Maximal subgroups
20P05 Probabilistic methods in group theory

Software:
GAP Character Table Library; GAP; CTblLib; Magma

Full Text: DOI arXiv

References:

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