Let $G$ be a finite domain in $\mathbb{C}$ bounded by a Jordan curve $L$. Given a fixed point $z_0$ in $G$ let $f$ be the conformal map of $G$ onto a disk $\{ |w| < r \}$ such that $f(z_0) = 0$ and $f'(z_0) = 1$. Let $B_n$ be the unique polynomial of degree $\leq n$ (Bieberbach polynomial) for which the integral

$$ \int \int_G |P'(z)|^2 d\sigma $$

is minimal in the class of all polynomials $P$ of degree $\leq n$ with $P(z_0) = 0$, $P'(z_0) = 1$.

The author gives (without proofs) estimates of the expressions $\sup_{z \in G} |f(z) - B_n(z)|$ for some classes of domains $G$ such that $L = \partial G$ is a union of quasiconformal arcs $L_1, \ldots, L_m$ meeting under zero inner angles. As a special case of his results the author obtains the following S. N. Mergelyan’s theorem [Trudy Mat. Inst. Steklov. 37, 92 p. (1951; Zbl 0045.353)]: If $L = \partial G$ is a Jordan curve with continuously turning tangent then for every $\epsilon > 0$

$$ \sup_{z \in G} |f(z) - B_n(z)| \leq C(\epsilon)n^{\epsilon^-}. $$

Reviewer: J. Siciak

MSC:

30E10 Approximation in the complex plane
30C20 Conformal mappings of special domains

Keywords:

conformal; Bieberbach polynomial