

Cholamjiak, Prasit; Shehu, Yekini

Inertial forward-backward splitting method in Banach spaces with application to compressed sensing. (English) Zbl 07088749

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Summary: We propose a Halpern-type forward-backward splitting with inertial extrapolation step for finding a zero of the sum of accretive operators in Banach spaces. Strong convergence of the sequence of iterates generated by the method proposed is obtained under mild assumptions. We give some numerical results in compressed sensing to validate the theoretical analysis results. Our result is one of the few available inertial-type methods for zeros of the sum of accretive operators in Banach spaces.

MSC:

47H05 Monotone operators and generalizations

Cited in 3 Documents

47J20 Variational and other types of inequalities involving nonlinear operators (general)

47J25 Iterative procedures involving nonlinear operators

Keywords:

[inertial term](#); [forward-backward splitting](#); [inclusion problem](#); [strong convergence](#); [Banach space](#)

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