de Jong, Peter

A central limit theorem for generalized multilinear forms. (English) Zbl 0709.60019

J. Multivariate Anal. 34, No. 2, 275-289 (1990).

Let X_i, $i \geq 1$, be independent random variables, and, for each $I \subset \{1, 2, ..., n\}$, let W_I be a function of $\{X_i, i \in I\}$. Suppose further that the W_I have mean zero and finite fourth moments, and that $\mathbb{E}W_I W_J = 0$ if $I \neq J$. Such random variables arise naturally in the decomposition of *W. Hoeffding* [Ann. Math. Stat. 19, 293-325 (1948; Zbl 0032.04101)].

The author is interested in conditions under which the d-homogeneous sum $W = \sum_{|I|=d} W_I$, standardized to have variance 1, converges to $N(0,1)$ as $n \to \infty$. This is not necessarily to be expected: if $d \geq 2$, a weighted mixture of centred chi-squared distributions is usual, but the setting is general enough to allow the possibility that all the weights become small as n increases. The main result is that under an asymptotic negligibility condition, it is sufficient that $\mathbb{E}W^4 \to 3$.

Reviewer: A.D.Barbour

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60F05 Central limit and other weak theorems

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