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**On multivariate Hermitian quadratic forms.** (English) Zbl 07095829

*Math. Comput. Sci.* 13, No. 1-2, 79-93 (2019)

**Summary:** Multivariate Hermitian quadratic forms play an important role in the real quantifier elimination algorithm based on the computation of comprehensive Gröbner systems introduced by V. Weispfenning and further improved by us. Our algorithm needs the computation of a certain type of saturation ideal in a parametric polynomial ring. In this paper, we study multivariate Hermitian quadratic forms in more detail and show several facts which have special importance in a parametric polynomial ring. Our results enable us to have an efficient method to compute the saturation ideal, which brings us a drastic improvement of our real quantifier elimination software.

**MSC:**

12D99 Real and complex fields

Cited in 1 Document

**Keywords:**

quantifier elimination; real closed field; comprehensive Gröbner system

**Software:**

QEPCAD; PGB; REDLOG; CGSQE

**Full Text:** [DOI](#)

**References:**

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