Campos, L. M. B. C.
On the solution of some simple fractional differential equations. (English) Zbl 0711.34019
Int. J. Math. Math. Sci. 13, No. 3, 481-496 (1990).

The aim of the author is to show that the classical elementary methods used to solve the ordinary differential equations with constant coefficients or the equations of Euler type can also be used to solve some functional equations in which the fundamental operator consists of a generalization of the classical concept of derivative, namely:

$$\frac{D^\nu F}{Dz^\nu} = \frac{\Gamma(1+\nu)}{2\pi i} \int_\infty^{(z+)} \frac{F(\zeta)d\zeta}{(\zeta - z)^{\nu+1}}, \quad \nu \neq -1, -2, \dots \quad ,$$

$$\frac{D^\nu F}{Dz^\nu} = \frac{1}{\Gamma(-\nu)} \int_\infty^{z} \frac{F(x)dx}{(z - x)^{\nu+1}}, \quad Re\nu < 0.$$

Notice that $D^n F/Dz^n$ is exactly equal to $F^{(n)}(z)$ when $n \geq 0$ is an integer and F(z) is regular near the point z while for $n = -1, -2, \dots$ it becomes equal to the n-times repeated integral of F(z). The generalization made by the author relies upon the fact that $D^\nu(e^{az})/Dz^\nu = a^\nu e^{az}$, which permits to write the solution of an equation of the form $\sum A_m D^{\nu_m} F/Dz^{\nu_m} = 0$ as $F(z) = \sum_m \sum_\ell C_{\ell m} z^\ell e^{a_m z}$. Similar results concerning the equations of Euler type as well as those with forcing terms are also given.

Reviewer: M.Idemen

MSC:

34A99	General theory for ordinary differential equations
34B30	Special ordinary differential equations (Mathieu, Hill, Bessel, etc.)
26A33	Fractional derivatives and integrals
45D05	Volterra integral equations
45J05	Integro-ordinary differential equations

Keywords:

Euler differential equation; constant coefficients

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