**Swartz, Charles W.**

**Orlicz-Pettis theorems for operators.** (English) Zbl 0711.47024

Southeast Asian Bull. Math. 12, No. 1, 31-38 (1988).

If (E,$\tau$) is a topological vector space, a series $\sum x_k$ in E is said to be $\tau$ s.s. (subseries) convergent if, for each subsequence $\{x_{n_k}\}$ of $\{x_k\}$, the subseries $\sum x_{n_k}$ is $\tau$-convergent to an element of E. The classical Orlicz-Pettis theorem states that if a series $\sum x_k$ in a normed space is s.s. convergent in the weak topology, then it is also s.s. convergent in the norm topology. In the paper under review several versions of the Orlicz- Pettis theorem for spaces of operators are considered.

Let X, Y be normed spaces and denote the space of bounded linear operators from X into Y by L(X,Y). If $A \subseteq L(X,Y)$, the notation $L_A(X,Y)$ is used for the space L(X,Y) equipped with the topology of uniform convergence on the zero sequences in X with respect to the weakest topology on X such that all the maps in A are continuous. For instance, it is proved that if $\sum_j T_j$ is s.s. convergent in the weak operator topology, then it is s.s. convergent in $L_A(X,Y)$, where $A = L(X,Y)$. In particular, it follows that weak operator s.s. convergence of a series in L(X,Y) implies s.s. convergence in the topology of uniform convergence on the precompact subsets of X. In general, weak operator s.s. convergence in L(X,Y) does not imply norm s.s. convergence - in fact, if weak operator and norm s.s. convergence coincide in L(X,Y), where X has an unconditional Schauder basis, then $L(X,Y) = K(X,Y)$ (compact operators) and $X'$ contains no subspace isomorphic to $\ell^\infty$. The final result of the paper is a generalization of *N. Kalton*'s Orlicz-Pettis result for compact operators [cf. Math. Ann. 208, 267-278 (1974; Zbl 0266.47038)] from which a "converse" of the last result follows: If X is a Banach space with unconditional Schauder basis such that $X'$ contains no subspace isomorphic to $\ell^\infty$, then $K(X,Y) = L(X,Y)$ if and only if weak operator s.s. convergence and norm s.s. convergence coincide.

**MSC:**

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| 47L05 | Linear spaces of operators |
| 46A35 | Summability and bases in topological vector spaces |
| 46B20 | Geometry and structure of normed linear spaces |



**Keywords:**

subseries convergent; Orlicz-Pettis theorem for spaces of operators; unconditional Schauder basis