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Multiplicative de Rham theorems for relative and intersection space cohomology. (English)

Zbl 1431.55006
J. Singul. 19, 97-130 (2019).

The authors "construct an explicit de Rham isomorphism relating the cohomology rings of Banagl's de Rham and spatial approach to intersection space cohomology for stratified pseudomanifolds with isolated singularities." Banagl assigned intersection spaces to certain classes of stratified spaces by a process of spatial homology truncation [*M. Banagl*, Intersection spaces, spatial homology truncation, and string theory. Dordrecht: Springer (2010; Zbl 1219.55001)]. These intersection spaces are cell complexes whose ordinary rational homology satisfies generalized Poincaré duality. Their cohomology is not isomorphic to intersection cohomology and, in contrast to the latter, intersection space cohomology is naturally equipped with perversity-internal cup-products. "De Rham Theorems for intersection space cohomology are given in [*M. Banagl*, J. Differ. Geom. 104, No. 1, 1–58 (2016; Zbl 1359.57016)] for pseudomanifolds with isolated singularities and in [*J. T. Essig*, About a de Rham complex describing intersection space cohomology in a non-isolated singularity case. University of Heidelberg (Master's thesis) (2012)] for pseudomanifolds of depth one with product link bundles."

However, it remains unclear whether the de Rham isomorphisms constructed in these papers respect the multiplicative structure and this is the question addressed in paper on hand. Its main result establishes an isomorphism of the cohomology rings in the case of isolated singularities. On the way the authors prove the de Rham Theorem for cohomology rings of pairs of smooth manifolds.

Reviewer: Beatrice Bleile (Armidale)

MSC:

55N33 Intersection homology and cohomology in algebraic topology
55N30 Sheaf cohomology in algebraic topology
14J17 Singularities of surfaces or higher-dimensional varieties
58A10 Differential forms in global analysis
58A12 de Rham theory in global analysis
57P10 Poincaré duality spaces
14J33 Mirror symmetry (algebro-geometric aspects)

Keywords:

singularity; stratified space; pseudomanifold; Poincaré duality; intersection space cohomology; intersection cohomology; sheaf theory; de Rham theorem; relative de Rham theorem; differential forms; cellular cup products; cup products on cochains

Full Text: DOI arXiv

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