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A note of F-topologies. (English) Zbl 0712.54001

Let \( X \) be a set and let \( 2^X \) denote its power set. A mapping \( u : 2^X \to 2^X \) is called an F-topology on \( X \) if 1) \( u\phi = \phi \); 2) \( A \subseteq uA \); 3) \( A \subseteq B \Rightarrow uA \subseteq uB \); and 4) \( u(uA) = uA \). Recall that any transitive binary relation on a set \( S \) is called a quasi-order on \( S \). We denote by \( A(X) \) the set of all quasi-orders \( \rho \) on \( 2^X \) satisfying the additional conditions: i) \( B \subseteq A \Rightarrow A \rho B \); ii) \( \phi \rho A \Rightarrow A = \phi \); and iii) if \( A \in 2^X \) and \( (B_i)_{i \in I} \) is a family in \( 2^X \) such that \( A \rho B_i \) for all \( i \in I \), then \( A \rho \cup_{i \in I} B_i \). Now the main result of the paper under review can be stated as follows: Theorem. Let \( \mathcal{B} \) be a cover of \( X \) and let \( u \) be an F-topology on \( X \). Then \( \mathcal{B} \) is an open base of \( u \) if and only if, for each pair of sets \( A, B \in 2^X \), there holds

\[
B \subseteq uA \Leftrightarrow (\forall C)(C \in \mathcal{B} \text{ and } A \subseteq X \setminus C \Rightarrow B \subseteq X \setminus C).
\]

Corollary. Let \( \rho \) be a binary relation on \( 2^X \). Then \( u \in A(X) \) if and only if there exists a cover \( \mathcal{B} \) of \( X \) such that, for each pair of sets \( A, B \in 2^X \), there holds

\[
A \rho B \Leftrightarrow (\forall C)(C \in \mathcal{B} \text{ and } A \subseteq X \setminus C \Rightarrow B \subseteq X \setminus C).
\]

Reviewer: P. Morales

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