Šlapal, Josef
A note of F-topologies. (English) Zbl 0712.54001
Math. Nachr. 141, 283-287 (1989).

Let X be a set and let 2^X denote its power set. A mapping $u : 2^X \to 2^X$ is called an F-topology on X if 1) $u\phi = \phi$; 2) $A \subseteq uA$; 3) $A \subseteq B \Rightarrow uA \subseteq uB$; and 4) $u(uA) = uA$. Recall that any transitive binary relation on a set S is called a quasi-order on S. We denote by A(X) the set of all quasi-orders ρ on 2^X satisfying the additional conditions: i) $B \subseteq A \Rightarrow A\rho B$; ii) $\phi\rho A \Rightarrow A = \phi$; and iii) if $A \in 2^X$ and $(B_i)_{i\in I}$ is a family in 2^X such that $A\rho B_i$ for all $i \in I$, then $A\rho \cup_{i\in I} B_i$. Now the main result of the paper under review can be stated as follows: Theorem. Let $\mathcal{B}$ be a cover of X and let u be an F-topology on X. Then $\mathcal{B}$ is an open base of u if and only if, for each pair of sets $A, B \in 2^X$, there holds

$$B \subseteq uA \Leftrightarrow (\forall C)(C \in \mathcal{B} \text{ and } A \subseteq X \setminus C \Leftrightarrow B \subseteq X \setminus C).$$

Corollary. Let ρ be a binary relation on 2^X. Then $u \in A(X)$ if and only if there exists a cover $\mathcal{B}$ of X such that, for each pair of sets $A, B \in 2^X$, there holds

$$A\rho B \Leftrightarrow (\forall C)(C \in \mathcal{B} \text{ and } A \subseteq X \setminus C \Rightarrow B \subseteq X \setminus C).$$

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