Eremenko, A. Eh.; **Lyubich, M. Yu.**
Dynamics of analytic transformations. (Russian) Zbl 0712.58036
Algebra Anal. 1, No. 3, 1-70 (1989).

This is a comprehensive survey paper on iterations of holomorphic maps in dimension 1. In Chapter 1, §3, classical local theorems are recalled: dynamics close to a (super) attracting or neutral rational fixed point. Also the linearization theory for irrational neutral points (Bryuno and recent ones of Yoccoz) is mentioned. In §4 iterations of holomorphic maps on hyperbolic Riemann surfaces are discussed. In particular in the case where the surface is the unit disc in $\mathbb{C}$ but one does not assume the map extends continuously to the boundary, the Denjoy-Wolff theory is mentioned.

Chapter 2 is entitled: Topological dynamics of rational endomorphisms. In § 1 the classical properties of Julia set are discussed. §2 is devoted to the complement of the Julia set, called the Fatou set, and its connected components are classified. In particular a proof of Sullivan's theorem: the nonexistence of a wandering component, is outlined. §3 is devoted to endomorphisms expanding on the Julia sets.

In §4 boundaries of Fatou components are discussed, in particular the dichotomy: analytic or no tangent. (The stronger theorem follows from the reviewer, *M. Urbański* and *A. Zdunik* [Ann. Math. (2) 130, No.1, 1-40 (1989; Zbl 0703.58036)], *M. Urbański* [Stud. Math. 97, No.3, 167-188 (1991; Zbl 0727.58024) and *A. Zdunik* [Invent. Math. 99, No.3, 627-649 (1990; Zbl 0820.58038)]; Trans. Am. Math. Soc.326, No.2, 633-652 (1991; Zbl 0766.30018)].) Namely the boundary of a basin of a sink is either analytic or of Hausdorff dimension > 1. Following Urbański [op.cit.] the same dichotomy holds for the boundary of a basin of a neutral periodic point if the boundary is disjoint, except for the point, from the ω-limit set of critical points.

§5 is devoted to Thurston's theory giving a criterion for a critically finite branched covering mapping of $\bar{\mathbb{C}}$ of the existence of an isotopic (relative critical values) rational map. For hyperbolic orbifolds uniqueness is claimed. Critically finite endomorphisms with parabolic orbifolds are discussed.

In §6 the Mañé-Sad-Sullivan λ-lemma is formulated including the generalized version of Sullivan-Thurston. This gives the genericity of structurally stable endomorphisms. Finally, McMullen's theorem about the nonexistence of a rational iterational algorithm for finding roots of polynomials of degree ≥ 4 is stated.

§7 is devoted to the Douady-Hubbard theory of the Mandelbrot set.

§§8 and 9 are devoted to the quasiconformal mappings technique. In particular, Sullivan's theorem that the space of rational endomorphisms q.c. conjugate to f is $T(f)/Mod(f)$ ($T(f)$ is a Teichmüller space, $Mod(f)$ is a modular group) is formulated. Douady-Hubbard polynomial-like maps are discussed. Finally Shishikura's q.c. surgery technique allowing construction of holomorphic maps by glueing them from pieces is explained. This gives sharp estimates for the number of Fatou periodic components.

In §10 there are analogies with Kleinian groups, in §11 the classical (Ritt) and new results on pairs of commuting rational maps.

Chapter 3 is devoted to measurable dynamics of rational maps. In §1 the theory of Gibbs measures in the expanding case is sketched. In §2 one looks from the point of view of 2-dimensional Lebesgue measure.

In §3 the $HD(\mu) = h_\mu(f)/\chi_\mu(f)$ formula for positive entropy measure μ is discussed. In §4 the measure with maximal entropy $\mu_{\max}$ and the respective Perron-Frobenius-Ruelle operator are considered. (Recently, papers on this operator for an arbitrary Hölder-continuous function by Denker, Urbański and the reviewer have appeared [*M. Denker* and *M. Urbański*, Nonlinearity 4, No.1, 103-134 (1991; Zbl 0718.58035); the reviewer, Bol. Soc. Bras. Mat., Nova Sér. 20, No.2, 95-125 (1990; Zbl 0723.58030)].) §5 is devoted to harmonic measures on the boundary of a basin of attraction to a sink and its relation to Hausdorff measures. Finally the authors discuss Zdunik's result [op. cit., 1990] that either Hausdorff dimension $HD(\mu_{\max}) < HD(\text{Julia set})$ (more precisely $< \sup(HD(\mu))$, supremum being taken over all probability ergodic measures of positive entropy) or f is critically finite with parabolic orbifold.

Chapter 4 is devoted to iterations of entire functions: the author's and Baker's theory.

MSC:

37J50	Action-minimizing orbits and measures (MSC2010)
37J10	Symplectic mappings, fixed points (dynamical systems) (MSC2010)
37-02	Research exposition (monographs, survey articles) pertaining to dynamical systems and ergodic theory
30D05	Functional equations in the complex plane, iteration and composition of analytic functions of one complex variable

Keywords:

survey; iterations of holomorphic maps; Topological dynamics; Julia set; Fatou set