Summary: The Univalence axiom, due to Vladimir Voevodsky, is often taken to be one of the most important discoveries arising from the Homotopy Type Theory (HoTT) research programme. It is said by Steve Awodey that Univalence embodies mathematical structuralism, and that Univalence may be regarded as ‘expanding the notion of identity to that of equivalence’. This article explores the conceptual, foundational and philosophical status of Univalence in Homotopy Type Theory. It extends our Types-as-Concepts interpretation of HoTT to Universes, and offers an account of the Univalence axiom in such terms. We consider Awodey’s informal argument that Univalence is motivated by the principle that reasoning should be invariant under isomorphism, and we examine whether an autonomous and rigorous justification along these lines can be given. We consider two problems facing such a justification. First, there is a difference between equivalence and isomorphism and Univalence must be formulated in terms of the former. Second, the argument as presented cannot establish Univalence itself but only a weaker version of it, and must be supplemented by an additional principle. The article argues that the prospects for an autonomous justification of Univalence are promising.

MSC:
03-XX Mathematical logic and foundations

Keywords:
homotopy type theory; equivalence; isomorphism; univalence; awodey; structuralism; universes

Software:
GitHub; HoTT

Full Text: DOI

References:


This reference list is based on information provided by the publisher or from digital mathematics libraries. Its items are heuristically matched to zbMATH identifiers and may contain data conversion errors. It attempts to reflect the references listed in the original paper as accurately as possible without claiming the completeness or perfect precision of the matching.