Bojarski, B.

On Gurov-Reshetnyak classes. (English) Zbl 0713.30018

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Let w(x) be a fixed weight function satisfying the strong doubling condition and let $d\mu = wdx$. The following result is proved. Let $f : Q_0 \to \mathbb{R}^m$ (Q_0 is a cube in R^n) satisfy, for some $\epsilon > 0$, some $q \geq 1$ and for each cube Q dyadic with respect to Q_0 the inequality

$$(1) \quad ((average) \quad \int_Q |f - f_{Q,w}|^q d\mu)^{1/q} \leq \epsilon \quad (average) \quad \int_Q |f| d\mu,$$

where

$$f_{Q,w} = \frac{1}{\mu(Q)} \int_Q f d\mu = (average) \quad \int_0 f d\mu.$$

Then for sufficiently small $\epsilon, \epsilon \leq \epsilon_0(n,q)$, there exists a constant C_0, depending on n, q and the constants involved in the doubling condition only such that f belongs to $L^p(Q_0)$ for every p, $q \leq p < C_0/\epsilon$. Moreover, for these values of p, the estimate

$$((average) \quad \int_Q |f - f_{Q,w}|^p d\mu)^{1/p} \leq C_1 \quad (average) \quad \int_Q |f| d\mu$$

holds, with the constant C_1 depending on q,n and the constants of the doubling condition only, for each dyadic cube $Q \subset Q_0$. Interesting applications of this result to reverse Hölder inequalities are given. In particular: let $B^p_q(K)$ $(p > q, K > 1)$ be the family of all functions satisfying the reverse Hölder inequality $\|f\|_{Q,q} \leq K\|f\|_{Q,q}$ for each cube Q contained in a fixed cube Q_0 in R^n, and denote by I(K,n,p,q) the optimal integrability exponent for all functions in $B^p_q(K)$; the author gives some information on the asymptotic of I(K,n,p,q) for $K \to 1$. Finally, a weak form of inequality (1) is investigated (Q is replaced by σ Q, $0 < \sigma < 1$, in the left hand side of (1)).

Reviewer: G.Porru

MSC:

30C65 Quasiconformal mappings in $\mathbb{R}^n$, other generalizations

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reverse Hölder inequalities