The authors investigate the minimax IMSE for nonparametric estimates of a regression function f in a Sobolev space. The regression model is: $Y_i = f(t_i) + \epsilon_i$, $1 \leq i \leq n$, with a fixed design (t_i) distributed according to a density g on [0,1] and with independent error variables $\{\epsilon_i\}$ with $E(\epsilon_i) = 0$. In their main result they give sharp lower bounds for

$$\lim_{n \to \infty} \inf_{\hat{f}} \sup_{f,\Pi} n^{2m/2m+1} E_\Pi \|\hat{f}_n - f\|_2^2,$$

where the infimum is taken over all estimates $\hat{f}$ and the supremum is taken over all f in the Sobolev-space $W_2^m(P)$ and all probability distributions Π of $(\epsilon_1, ..., \epsilon_n)$ with components which are in a shrinking neighborhood of a fixed distribution and have bounded fourth moment. It is shown that $\Delta \geq c(m, \sigma^2, g, P)$ with an explicit constant c.

This generalizes the case of normal error variables which was treated by *M. Nußbaum* [ibid. 13, 984-997 (1985; Zbl 0596.62052)]. Furthermore, the optimality of c, linear estimates, localized bounds and adaptive smoothing are discussed.

Reviewer: U.Stadtmüller

MSC:

62G07	Density estimation
62G20	Asymptotic properties of nonparametric inference
62C20	Minimax procedures in statistical decision theory

Cited in **1** Review
Cited in **8** Documents

Keywords:

integrated mean square error; nonnormal case; lower asymptotic risk bound; nonparametric regression; asymptotic minimax; smoothness ellipsoid; location model; shrinking Hellinger neighborhoods; adaptive bandwidth choice; experimental design; robust smoothing; L2-risk; minimax IMSE; independent error variables; Sobolev-space; bounded fourth moment; optimality; linear estimates; localized bounds; adaptive smoothing

Full Text: DOI