Ho, Ky; **Kim, Yun-Ho**

A-priori bounds and multiplicity of solutions for nonlinear elliptic problems involving the fractional $p(\cdot)$-Laplacian. (English) Zbl 1425.35041

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Summary: We obtain fundamental imbeddings for fractional Sobolev spaces with variable exponents, which are a generalization of the well-known fractional Sobolev spaces. As an application, we obtain a-priori bounds and multiplicity of solutions to some nonlinear elliptic problems involving the fractional $p(\cdot)$-Laplacian.

MSC:

35J60 Nonlinear elliptic equations
35J20 Variational methods for second-order elliptic equations
35B45 A priori estimates in context of PDEs
35J92 Quasilinear elliptic equations with p-Laplacian
35R11 Fractional partial differential equations
46E35 Sobolev spaces and other spaces of "smooth" functions, embedding theorems, trace theorems

Keywords:

fractional p-Laplacian; $p(\cdot)$-Laplacian; fractional Sobolev spaces with variable exponents; a-priori bounds; de Giorgi iteration; variational methods

Full Text: DOI arXiv

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