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**Isotropic metric spaces.** (English. Russian original) Zbl 0714.54031


A purely metric variant of the classical Schur theorem in Riemannian geometry is presented. In an arbitrary metric space it is possible to define the isotropic Riemannian curvature at a point $P$ as a limit $K(P) = \lim_{T \to P} d(T)/s(T)$ (if it exists) where $T$ is a triangle (shrinking to $P$ in the limit), $d(T)$ denotes the excess and $s(T)$ “Euclidean area” of the triangle $T$. A metric space $M$ is said to be isotropic if the isotropic Riemannian curvature exists at each point of $M$. The main result of the paper is the proof of the following theorem: Suppose that $M$ is a locally compact metric space with intrinsic metric and with Menger-Urysohn dimension greater than two. Assume that the shortest path is locally extendible in $M$ and $M$ is an isotropic metric space. Then $M$ is isometric to the space of constant curvature.

Furthermore, if $M$ is a metric space with intrinsic metric, that is a topological manifold of finite dimension greater than two. Assume that the Wald curvature $K'_W(P)$ exists at each point $P \in M$. Then $M$ is isometric to a space of constant curvature.

Reviewer: J.Bureš

**MSC:**

- 54E45 Compact (locally compact) metric spaces
- 53C20 Global Riemannian geometry, including pinching
- 53A10 Minimal surfaces in differential geometry, surfaces with prescribed mean curvature

**Keywords:**

isotropic Riemannian curvature; isotropic metric space; space of constant curvature