

Rupel, Dylan C.

Rank two non-commutative Laurent phenomenon and pseudo-positivity. (English)

Zbl 1436.16018

Algebr. Comb. 2, No. 6, 1239-1273 (2019).

Summary: We study polynomial generalizations of the Kontsevich automorphisms acting on the skew-field of formal rational expressions in two non-commuting variables. Our main result is the Laurentness and pseudo-positivity of iterations of these automorphisms. The resulting expressions are described combinatorially using a generalization (studied in our work [“Greedy bases in rank 2 generalized cluster algebras”, Preprint, [arXiv:1309.2567](https://arxiv.org/abs/1309.2567)]) of the combinatorics of compatible pairs in a maximal Dyck path developed by *K. Lee* et al. [*Sel. Math., New Ser.* 20, No. 1, 57–82 (2014; [Zbl 1295.13031](https://zbmath.org/journal/Sel-Math-New-Ser-20-1-57-82-2014))].

By specializing to quasi-commuting variables we obtain pseudo-positive expressions for rank 2 quantum generalized cluster variables. In the case that all internal exchange coefficients are zero, this quantum specialization provides a positive combinatorial construction of counting polynomials for Grassmannians of submodules in exceptional representations of valued quivers with two vertices.

MSC:

[16G20](#) Representations of quivers and partially ordered sets

[05E10](#) Combinatorial aspects of representation theory

[13F60](#) Cluster algebras

Cited in 1 Document

Keywords:

non-commutative cluster; Kontsevich automorphism; maximal Dyck path; quiver Grassmannian

Full Text: [DOI](#)

References:

- [1] Bai, Liqian; Chen, Xueqing; Ding, Ming; Xu, Fan, A quantum analogue of generalized cluster algebras (2016) · [Zbl 1408.16008](#)
- [2] Berenstein, Arkady; Retakh, Vladimir, A Short Proof of Kontsevich Cluster Conjecture, *C. R. Math. Acad. Sci. Paris*, 349, 3-4, 119-122 (2011) · [Zbl 1266.16026](#)
- [3] Caldero, Philippe; Reineke, Markus, On the quiver Grassmannian in the acyclic case, *J. Pure Appl. Algebra*, 212, 11, 2369-2380 (2008) · [Zbl 1153.14032](#)
- [4] Chekhov, Leonid; Shapiro, Michael, Teichmüller Spaces of Riemann Surfaces with Orbifold Points of Arbitrary Order and Cluster Variables, *Int. Math. Res. Not.*, 10, 2746-2772 (2014) · [Zbl 1301.30042](#)
- [5] Di Francesco, Philippe; Kedem, Rinat, Discrete Non-Commutative Integrability: Proof of a Conjecture of M. Kontsevich, *Int. Math. Res. Not.*, 21, 4042-4063 (2010) · [Zbl 1276.16025](#)
- [6] Fomin, Sergey; Zelevinsky, Andrei, Cluster Algebras I. Foundations, *J. Amer. Math. Soc.*, 15, 2, 497-529 (2002) · [Zbl 1021.16017](#)
- [7] Kontsevich, Maxim, Noncommutative identities
- [8] Lee, Kyungyong; Li, Li; Zelevinsky, Andrei, Greedy elements in rank 2 cluster algebras, *Selecta Math.*, 20, 1, 57-82 (2014) · [Zbl 1295.13031](#)
- [9] Lee, Kyungyong; Schiffler, Ralf, Proof of a Positivity Conjecture of M. Kontsevich on Non-Commutative Cluster Variables, *Compos. Math.*, 148, 6, 1821-1832 (2012) · [Zbl 1266.16027](#)
- [10] Rupel, Dylan, Greedy bases in rank 2 generalized cluster algebras · [Zbl 1371.16014](#)
- [11] Rupel, Dylan, On a quantum analog of the Caldero-Chapoton formula, *Int. Math. Res. Not.*, 14, 3207-3236 (2011) · [Zbl 1237.16013](#)
- [12] Rupel, Dylan, Proof of the Kontsevich non-commutative cluster positivity conjecture, *C. R. Math. Acad. Sci. Paris*, 350, 21-22, 929-932 (2012) · [Zbl 1266.16028](#)
- [13] Rupel, Dylan, Quantum cluster characters for valued quivers, *Trans. Amer. Math. Soc.*, 367, 10, 7061-7102 (2015) · [Zbl 1371.16014](#)
- [14] Usnich, Alexandr, Non-commutative Laurent phenomenon for two variables
- [15] Usnich, Alexandr, Non-commutative cluster mutations, *Dokl. Nats. Akad. Nauk Belarusi*, 53, 4, 27-29 (2009) · [Zbl 1267.16012](#)

- [16] Usnich, Alexandr, Action of the Cremona group on a non-commutative ring, *Adv. Math.*, 228, 4, 1863-1893 (2011) · [Zbl 1250.14008](#)

This reference list is based on information provided by the publisher or from digital mathematics libraries. Its items are heuristically matched to zbMATH identifiers and may contain data conversion errors. It attempts to reflect the references listed in the original paper as accurately as possible without claiming the completeness or perfect precision of the matching.