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Number of weak Galois-Weierstrass points with Weierstrass semigroups generated by two elements. (English) Zbl 1428.14055

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Let C be a non-singular projective curve of genus $g \geq 2$ over an algebraically closed field of characteristic 0. Take a point P on C. Its Weierstrass semigroup $H(P)$ is the set of non-negative integers n for which there exists a rational function f on C such that f has a pole of order n at P, and is regular away from P. The point P is a Galois Weierstrass point (GW point, in short), if $\Phi_{|aP|} : C \to \mathbb{P}^1$ is a Galois covering where a is the smallest positive integer of $H(P)$. Besides, P is said to be a weak Galois-Weierstrass point (weak GW point), if it is a Weierstrass point and there exists a Galois morphism $\varphi : C \to \mathbb{P}^1$ such that P is a total ramification point of φ.

The paper under review is devoted to study the number of weak GW points which satisfy that their Weierstrass semigroup $H(P)$ is generated by two positive integers, a and b, such that $\gcd(a, b) = 1$ and $2 < a < b - 1$. The main result of the article is Theorem 1.3. In its first part it is proved that the number of GW points P with $H(P) = \langle a, b\rangle$ is 0 or $b + 1$ if $b \equiv -1 \pmod a$, and it is 0 or 1 if $b \not\equiv -1 \pmod a$. For second and third parts of Theorem 1.3, let P be a weak GW point, and call $\mathrm{degGW}(P)$ the set of degrees of the Galois coverings of C totally ramified at P. Then, the number of weak GW points P with $H(P) = \langle a, b\rangle$ and $b \in \mathrm{degGW}(P)$ is 0 or 1, and there exists a weak GW point P with $H(P) = \langle a, b\rangle$, and $a, b \in \mathrm{degGW}(P)$ if and only if C is birationally equivalent to the curve $X^b = Y^a Z^{b-a} + Z^b$.

It is important to note that *M. Coppens* has obtained in [Abh. Math. Semin. Univ. Hamb. 89, No. 1, 1–16 (2019; Zbl. 07100734)] results which overlap partially the mentioned Theorem 1.3.

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MSC:

14H55 Riemann surfaces; Weierstrass points; gap sequences
14H50 Plane and space curves
14H30 Coverings of curves, fundamental group
20M14 Commutative semigroups

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