Dehestani, Haniye; **Ordokhani, Yadollah**; **Razzaghi, Mohsen**
Fractional-order Bessel functions with various applications. (English) Zbl 07144731
Appl. Math., Praha 64, No. 6, 637-662 (2019)

Summary: We introduce fractional-order Bessel functions (FBFs) to obtain an approximate solution for various kinds of differential equations. Our main aim is to consider the new functions based on Bessel polynomials to the fractional calculus. To calculate derivatives and integrals, we use Caputo fractional derivatives and Riemann-Liouville fractional integral definitions. Then, operational matrices of fractional-order derivatives and integration for FBFs are derived. Also, we discuss an error estimate between the computed approximations and the exact solution and apply it in some examples. Applications are given to three model problems to demonstrate the effectiveness of the proposed method.

MSC:

34A08 Fractional ordinary differential equations
65M70 Spectral, collocation and related methods for initial value and initial-boundary value problems involving PDEs
65L70 Error bounds for numerical methods for ordinary differential equations

Keywords:

fractional-order Bessel functions; fractional operational matrix; error estimation

Full Text: DOI

References:

[1] Agarwal, R.; O'Regan, D.; Hristova, S., Stability of Caputo fractional differential equations by Lyapunov functions, Appl. Math., Praha 60 (2015), 653-676· doi:10.1007/s10492-015-0116-4

[2] Baillie, R. T., Long memory processes and fractional integration in econometrics, J. Econom. 73 (1996), 5-59· doi:10.1016/0304-4076(95)01732-1

[3] Bhrawy, A. H.; Alhamed, Y.; Baleanu, D.; Al-Zahrani, A., New spectral techniques for systems of fractional differential equations using fractional-order generalized Laguerre orthogonal functions, Fract. Calc. Appl. Anal. 17 (2014), 1138-1157· doi:10.2478/s13540-014-0218-9

[4] Bohannan, G. W., Analog fractional order controller in temperature and motor control applications, J. Vib. Control 14 (2008), 1487-1498· doi:10.1177/1077546307087435

[5] Caputo, M., Linear models of dissipation whose \(Q\) is almost frequency independent. II, Geophys. J. R. Astron. Soc. 13 (1967), 529-539· doi:10.1111/j.1365-246X.1967.tb02303.x

[6] Chen, Y.; Sun, Y.; Liu, L., Numerical solution of fractional partial differential equations with variable coefficients using generalized fractional-order Legendre functions, Appl. Math. Comput. 244 (2014), 847-858· doi:10.1016/j.amc.2014.07.050

[7] Chen, X.; Wang, L., The variational iteration method for solving a neutral functional differential equation with proportional delays, Comput. Math. Appl. 59 (2010), 2696-2702· doi:10.1016/j.camwa.2010.01.037

[8] Dehestani, H.; Ordokhani, Y.; Razzaghi, M., Fractional-order Legendre-Laguerre functions and their applications in fractional partial differential equations, Appl. Math. Comput. 336 (2018), 433-453· doi:10.1016/j.amc.2018.05.017

[9] Dehestani, H.; Ordokhani, Y.; Razzaghi, M., On the applicability of Genocchi wavelet method for different kinds of fractional-order differential equations with delay, Numer. Linear Algebra Appl. 26 (2019), Article ID e2259, 29 pages· doi:10.1002/nla.2259

[10] Dehestani, H.; Ordokhani, Y.; Razzaghi, M., Application of the modified operational matrices in multiterm variable-order time-fractional partial differential equations, (to appear) in Math. Methods Appl. Sci., 18 pages· doi:10.1002/mma.5840

[11] Doha, E. H.; Bhrawy, A. H.; Baleanu, D.; Hafez, R. M., A new Jacobi rational-Gauss collocation method for numerical solution of generalized pantograph equations, Appl. Numer. Math. 77 (2014), 43-54· doi:10.1016/j.apnum.2013.11.003

[12] Engheta, N., On fractional calculus and fractional multipoles in electromagnetism, IEEE Trans. Antennas Propag. 44 (1996), 554-566· doi:10.1109/8.489308

[13] Grosswald, E., Bessel Polynomials, Lecture Notes in Mathematics 698, Springer, Berlin (1978)· doi:10.1007/bfb0063135

[14] He, J., Approximate analytical solution for seepage flow with fractional derivatives in porous media, Comput. Methods Appl. Mech. Eng. 167 (1998), 57-68· doi:10.1016/S0045-7825(98)00108-X

[15] He, J., Nonlinear oscillation with fractional derivative and its applications, Proceedings of the International Conference on Vibrating Engineering, Dalian, 1998, pp. 288-291

[16] Iqbal, M. A.; Saeed, U.; Mohyud-Din, S. T., Modified Laguerre wavelets method for delay differential equations of fractional-

order, Egyptian J. Basic Appl. Sci. 2 (2015), 50-54· doi:10.1016/j.ejbas.2014.10.004

[17] Jafari, H.; Yousefi, S. A.; Firoozjaee, M. A.; Momani, S.; Khalique, C. M., Application of Legendre wavelets for solving fractional differential equations, Comput. Math. Appl. 62 (2011), 1038-1045· doi:10.1016/j.camwa.2011.04.024

[18] Kazem, S., Exact solution of some linear fractional differential equations by Laplace transform, Int. J. Nonlinear Sci. 16 (2013), 3-11

[19] Kazem, S.; Abbasbandy, S.; Kumar, S., Fractional-order Legendre functions for solving fractional-order differential equations, Appl. Math. Modelling 37 (2013), 5498-5510· doi:10.1016/j.apm.2012.10.026

[20] Kreyszig, E., Introductory Functional Analysis with Applications, John Wiley \& Sons, New York (1978)

[21] Kumar, P.; Agrawal, O. P., An approximate method for numerical solution of fractional differential equations, Signal Process. 86 (2006), 2602-2610· doi:10.1016/j.sigpro.2006.02.007

[22] Li, Y., Solving a nonlinear fractional differential equation using Chebyshev wavelets, Commun. Nonlinear Sci. Numer. Simul. 15 (2010), 2284-2292· doi:10.1016/j.cnsns.2009.09.020

[23] Li, X. Y.; Wu, B. Y., A continuous method for nonlocal functional differential equations with delayed or advanced arguments, J. Math. Anal. Appl. 409 (2014), 485-493· doi:10.1016/j.jmaa.2013.07.039

[24] Liu, F.; Anh, V.; Turner, I., Numerical solution of the space fractional Fokker-Planck equation, J. Comput. Appl. Math. 166 (2004), 209-219· doi:10.1016/j.cam.2003.09.028

[25] Mainardi, F., Fractional calculus: Some basic problems in continuum and statistical mechanics, Fractals and Fractional Calculus in Continuum Mechanics CISM Courses and Lectures 378, Springer, Vienna (1997), 291-348· doi:10.1007/978-3-7091-2664-6$_7$

[26] Mandelbrot, B., Some noises with \(1/f\) spectrum, a bridge between direct current and white noise, IEEE Trans. Inf. Theory 13 (1967), 289-298· doi:10.1109/TIT.1967.1053992

[27] Miller, K. S.; Ross, B., An Introduction to the Fractional Calculus and Fractional Differential Equations, John Wiley \& Sons, New York (1993)

[28] Moaddy, K.; Momani, S.; Hashim, I., The non-standard finite difference scheme for linear fractional PDEs in fluid mechanics, Comput. Math. Appl. 61 (2011), 1209-1216· doi:10.1016/j.camwa.2010.12.072

[29] Momani, S.; Al-Khaled, K., Numerical solutions for systems of fractional differential equations by the decomposition method, Appl. Math. Comput. 162 (2005), 1351-1365· doi:10.1016/j.amc.2004.03.014

[30] Momani, S.; Odibat, Z., Numerical approach to differential equations of fractional order, J. Comput. Appl. Math. 207 (2007), 96-110· doi:10.1016/j.cam.2006.07.015

[31] Oldham, K. B.; Spanier, J., The Fractional Calculus. Theory and Applications of Differentiation and Integration to Arbitrary Order, Mathematics in Science and Engineering 111, Academic Press, New York (1974)· doi:10.1016/S0076-5392(09)60219-8

[32] Parand, K.; Nikarya, M., Application of Bessel functions for solving differential and integro-differential equations of the fractional order, Appl. Math. Modelling 38 (2014), 4137-4147· doi:10.1016/j.apm.2014.02.001

[33] Parand, K.; Nikarya, M.; Rad, J. A., Solving non-linear Lane-Emden type equations using Bessel orthogonal functions collocation method, Celest. Mech. Dyn. Astron. 116 (2013), 97-107· doi:10.1007/s10569-013-9477-8

[34] Petráš, I., Fractional-order feedback control of a DC motor, J. Electr. Eng. 60 (2009), 117-128

[35] Podlubny, I., Fractional Differential Equations. An Introduction to Fractional Derivatives, Fractional Differential Equations, to Methods of Their Solution and Some of Their Applications, Mathematics in Science and Engineering 198, Academic Press, San Diego (1999)

[36] Rahimkhani, P.; Ordokhani, Y.; Babolian, E., Fractional-order Bernoulli wavelets and their applications, Appl. Math. Modelling 40 (2016), 8087-8107· doi:10.1016/j.apm.2016.04.026

[37] Rahimkhani, P.; Ordokhani, Y.; Babolian, E., Numerical solution of fractional pantograph differential equations by using generalized fractional-order Bernoulli wavelet, J. Comput. Appl. Math. 309 (2017), 493-510· doi:10.1016/j.cam.2016.06.005

[38] Rivlin, T. J., An Introduction to the Approximation of Functions, Dover Books on Advanced Mathematics, Dover Publications, New York (1981)

[39] Saeed, U.; Rehman, M. ur; Iqbal, M. A., Modified Chebyshev wavelet methods for fractional delay-type equations, Appl. Math. Comput. 264 (2015), 431-442· doi:10.1016/j.amc.2015.04.113

[40] Saeedi, H.; Moghadam, M. M.; Mollahasani, N.; Chuev, G. N., A CAS wavelet method for solving nonlinear Fredholm integro-differential equations of fractional order, Commun. Nonlinear Sci. Numer. Simul. 16 (2011), 1154-1163· doi:10.1016/j.cnsns.2010.05.036

[41] Tohidi, E.; Nik, H. Saberi, A Bessel collocation method for solving fractional optimal control problems, Appl. Math. Modelling 39 (2015), 455-465· doi:10.1016/j.apm.2014.06.003

[42] Wang, W.-S.; Li, S.-F., On the one-leg \(\theta \)-methods for solving nonlinear neutral functional differential equations, Appl. Math. Comput. 193 (2007), 285-301· doi:10.1016/j.amc.2007.03.064

[43] Yin, F.; Song, J.; Wu, Y.; Zhang, L., Numerical solution of the fractional partial differential equations by the two-dimensional fractional-order Legendre functions, Abstr. Appl. Anal. 2013 (2013), Article ID 562140, 13 pages· doi:10.1155/2013/562140

[44] Yuanlu, L.; Weiwei, Z., Haar wavelet operational matrix of fractional order integration and its applications in solving the fractional order differential equations, Appl. Math. Comput. 216 (2010), 2276-2285· doi:10.1016/j.amc.2010.03.063

[45] Yüzbaşi, Ş., Bessel Polynomial Solutions of Linear Differential, Integral and Integro-Differential Equations, M.Sc. Thesis, Graduate School of Natural and Applied Sciences, Mugla University, Kötekli (2009)

[46] Yüzbaşi, Ş., Numerical solutions of fractional Riccati type differential equations by means of the Bernstein polynomials, Appl.

Math. Comput. 219 (2013), 6328-6343· doi:10.1016/j.amc.2012.12.006

[47] Yüzbaşi, Ş.; Şahin, N.; Sezer, M., Numerical solutions of systems of linear Fredholm \hbox{integro}-differential equations with Bessel polynomial bases, Comput. Math. Appl. 61 (2011), 3079-3096· doi:10.1016/j.camwa.2011.03.097

[48] Zhang, X.; Tang, B.; He, Y., Homotopy analysis method for higher-order fractional \hbox{integro}–differential equations, Comput. Math. Appl. 62 (2011), 3194-3203· doi:10.1016/j.camwa.2011.08.032