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Complete mappings and incremental ratio in double-loops. (Italian. English summary)


Let $(K, +, .)$ be a double loop, i.e. $(K, +)$ and $(K^*, .)$ are loops, where $K^* = K \setminus \{0\}$, the neutral element in $(K, +)$. For a bijection $f: K \rightarrow K$ and a fixed element $a \in K$, the author defines the incremental ratio $f_a(x) = (f(x) - f(a))(x - a)^{-1}$, where $-y$ denotes the inverse w.r.t. “+”, and $y^{-1}$ denotes the inverse w.r.t. “.”. Denote $C(f) = \{ a \in K | \ f_a : K \setminus \{a\} \rightarrow K^* \text{ is a bijection} \}$. The main results in the paper are: Theorem 1. If $(K, +, .)$ is a double loop satisfying the conditions that $(K, +)$ has the right inverse property and there exists a bijection $f: K \rightarrow K$ with $C(f) \neq \emptyset$, then there exists a bijection $g$ of $K$ such that $0 \in C(g)$ and, equivalently, $(K^*, .)$ has complete mappings. (A complete mapping of $(K^*, .)$ is a bijection $\theta : K^* \rightarrow K^*$ such that $\eta : x \rightarrow x \cdot \theta(x)$ is also a bijection of $K^*$.) Theorem 2. If $(K, +, .)$ is a double loop with the right inverse property for $(K, +)$, having $4k + 3$ elements and $(K^*, .)$ has a subloop with $2k + 1$ elements, then $C(f) = \emptyset$ for a bijection $f$ of $K$. Theorem 3. An associative nearfield $(N, +, .)$ is a double loop if and only if it is finite and the group $(N^*, .)$ has only cyclic Sylow 2-subgroups. The author proves that $SL(2, q)$, for $q = 2^h$ and $h \geq 2$, and for $q \in \{5, 7, 11\}$, has complete mappings.

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