Saeli, Donato

Complete mappings and incremental ratio in double-loops. (Italian. English summary)

Zbl 0715.20045
Riv. Mat. Univ. Parma, IV. Ser. 15, 111-117 (1989).

Let $(K,+,.)$ be a double loop, i.e. $(K,+)$ and $(K^*,.)$ are loops, where K^* is $K\setminus\{0\}$, the neutral element in $(K,+)$. For a bijection f: $K\to K$ and a fixed element $a\in K$, the author defines the incremental ratio $f_a(x)=(f(x)-f(a))(x-a)^{-1}$, where -y denotes the inverse w.r.t. "+", and y^{-1} denotes the inverse w.r.t. ".". Denote $C(f)=\{a\in K|\ f_a:K\setminus\{a\}\to K^*$ is a bijection$\}$. The main results in the paper are: Theorem 1. If $(K,+,.)$ is a double loop satisfying the conditions that $(K,+)$ has the right inverse property and there exists a bijection f: $K\to K$ with C(f)$\neq\emptyset$, then there exists a bijection g of K such that $0\in C(g)$ and, equivalently, $(K^*,.)$ has complete mappings. (A complete mapping of $(K^*,.)$ is a bijection $\theta:K^*\to K^*$ such that $\eta:x\to x\cdot\theta(x)$ is also a bijection of K^*.) Theorem 2. If $(K,+,.)$ is a double loop with the right inverse property for $(K,+)$, having $4k+3$ elements and $(K^*,.)$ has a subloop with $2k+1$ elements, then $C(f)=\emptyset$ for a bijection f of K. Theorem 3. An associative nearfield $(N,+,.)$ is a double loop if and only if it is finite and the group $(N^*,.)$ has only cyclic Sylow 2-subgroups. The author proves that SL(2,q), for $q=2^h$ and $h\geq 2$, and for $q\in\{5,7,11\}$, has complete mappings.

Reviewer: M.Ştefănescu

MSC:

20N05	Loops, quasigroups
20D60	Arithmetic and combinatorial problems involving abstract finite groups
12K05	Near-fields

Keywords:

double loop; incremental ratio; right inverse property; complete mappings; associative nearfield; cyclic Sylow 2-subgroups