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A characterization of circle graphs in terms of multimatroid representations. (English)

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Summary: The isotropic matroid $M[IAS(G)]$ of a looped simple graph G is a binary matroid equivalent to the isotropic system of G. In general, $M[IAS(G)]$ is not regular, so it cannot be represented over fields of characteristic $\neq 2$. The ground set of $M[IAS(G)]$ is denoted $W(G)$; it is partitioned into 3-element subsets corresponding to the vertices of G. When the rank function of $M[IAS(G)]$ is restricted to subtransversals of this partition, the resulting structure is a multimatroid denoted $\mathcal{Z}_3(G)$. In this paper we prove that G is a circle graph if and only if for every field $\mathbb{F}$, there is an $\mathbb{F}$-representable matroid with ground set $W(G)$, which defines $\mathcal{Z}_3(G)$ by restriction. We connect this characterization with several other circle graph characterizations that have appeared in the literature.

MSC:

05B35 Combinatorial aspects of matroids and geometric lattices
52B40 Matroids in convex geometry (realizations in the context of convex polytopes, convexity in combinatorial structures, etc.)

Keywords:

circle graph; multimatroid; delta-matroid; isotropic system; local equivalence; matroid; regularity; representation; unimodular orientation

Full Text: Link arXiv

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