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Critical values and the determinant of periods. (English. Russian original) Zbl 0716.32024

Russ. Math. Surv. 44, No. 4, 209-210 (1989); translation from Usp. Mat. Nauk 44, No. 4(268), 235-236 (1989).

Consider the Pham polynomial $x_1^{k_1}+...+x_n^{k_n}$ and its deformation $f(x)=x_1^{k_1}+...x_n^{k_n}+\sum_{|m|<1}\lambda_m x_1^{m_1}...x_n^{m_n}$, where $|(m_1,...,m_n)|=m_1/k_1+...+m_n/k_n$ and $\lambda_m\in\mathbb{C}$. $X=\{x\in\mathbb{C}^n|f(x)=0\}$ has the homotopy type of a bouquet of μ (n-1)-dimensional spheres, where $\mu=(k_1-1)...(k_n-1)$. Consider the canonical base $\delta_1,...,\delta_\mu\in H_{n-1}(X,\mathbb{Z})$ and the base for the cohomology given by the forms ω_I/df, where $\omega_I=x_1^{i_1}...x_n^{i_n}dx_1\wedge...\wedge dx_n$, $0\le i_j\le k_j-2$.

The main result is a formula expressing $\det(\int_{\delta_j}\omega_I/df)$ as (n/2-1)-power of the product of the critical values of f, modulo the multiplication with a well determined constant. One asks for a similar formula when starting with an arbitrary quasi- homogeneous polynomial.

Other related results are also given, as part of a general principle; express the determinant of the matrix of periods in terms of the critical values of the equations defining the variety.

Reviewer: C.Bănică

MSC:

32S20 Global theory of complex singularities; cohomological properties
32G20 Period matrices, variation of Hodge structure; degenerations
58C25 Differentiable maps on manifolds
58K99 Theory of singularities and catastrophe theory

Keywords:

critical values; affine hypersurfaces; Pham polynomial; matrix of periods

Full Text: DOI